```
Amplitudes=[C0;Cn]
Angles=Dnangles(1:M);
Angles=Angles*(180/pi);
disp('Amplitudes Angles')
[Amplitudes Angles]
% To Plot the Fourier coefficients
k=0:length(Amplitudes)-1; k=k';
subplot(211),stem(k,Amplitudes)
subplot(212),stem(k,Angles)
ans =
    Amplitudes   Angles
    0.5043       0
    0.2446       -75.9622
    0.1251       -82.8719
    0.0837       -85.2317
    0.0629       -86.4175
    0.0503       -87.1299
    0.0419       -87.6048
    0.0359       -87.9437
    0.0314       -88.1977
    0.0279       -88.3949
```

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2. B. P. Lathi, *Signal Processing and Linear Systems*, Berkeley-Cambridge Press, Carmichael, CA, 1998.
3. P. L. Walker, *The Theory of Fourier Series and Integrals*, Wiley-Interscience, New York, 1986.
4. R. V. Churchill, and J. W. Brown, *Fourier Series and Boundary Value Problems*, 3rd ed., McGraw-Hill, New York, 1978.

PROBLEMS

2.1-1 Find the energies of the signals shown in Fig. P2.1-1. Comment on the effect on energy of sign change, time shifting or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

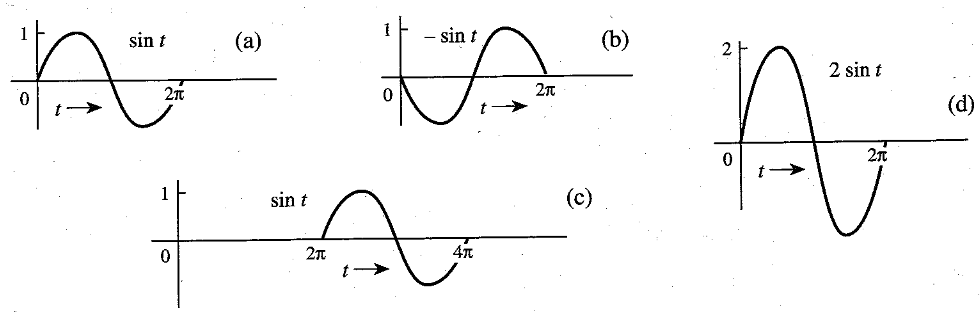


Figure P2.1-1

2.1-2 **(a)** Find E_x and E_y, the energies of the signals $x(t)$ and $y(t)$ shown in Fig. P2.1-2a. Sketch the signals $x(t) + y(t)$ and $x(t) - y(t)$ and show that the energies of either of these two signals are equal to $E_x + E_y$. Repeat the procedure for the signal pair of Fig. P2.1-2b.

(b) Repeat the procedure for the signal pair of Fig. P2.1-2c. Are the energies of the signals $x(t) + y(t)$ and $x(t) - y(t)$ identical in this case?

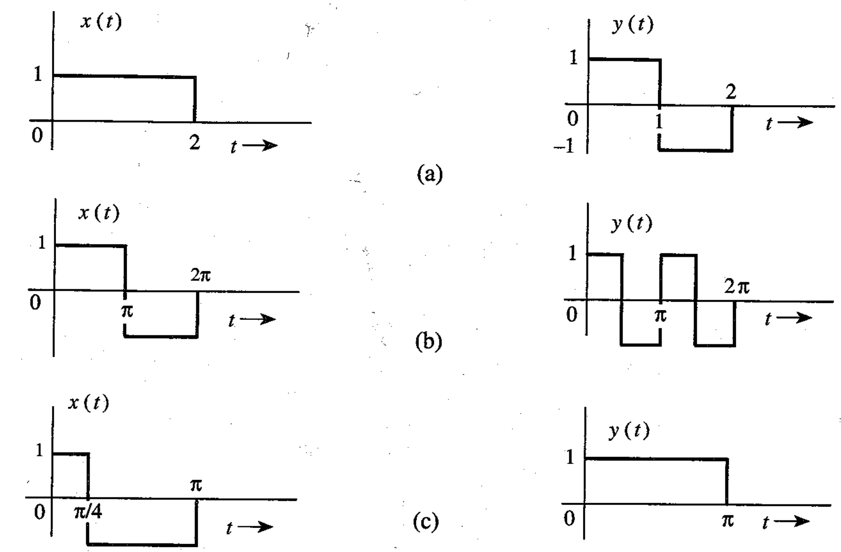


Figure P2.1-2

2.1-3 Redo Example 2.2a to find the power of a sinusoid $C \cos(\omega_0 t + \theta)$ by averaging the signal energy over one period $2\pi/\omega_0$ (rather than averaging over the infinitely large interval).

2.1-4 Show that if $\omega_1 = \omega_2$, the power of $g(t) = C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_2 t + \theta_2)$ is $[C_1{}^2 + C_2{}^2 + 2C_1C_2 \cos(\theta_1 - \theta_2)]/2$, which is not equal to $(C_1{}^2 + C_2{}^2)/2$.

2.1-5 Find the power of the periodic signal $g(t)$ shown in Fig. P2.1-5. Find also the powers and the rms values of: **(a)** $-g(t)$; **(b)** $2g(t)$; **(c)** $cg(t)$. Comment.

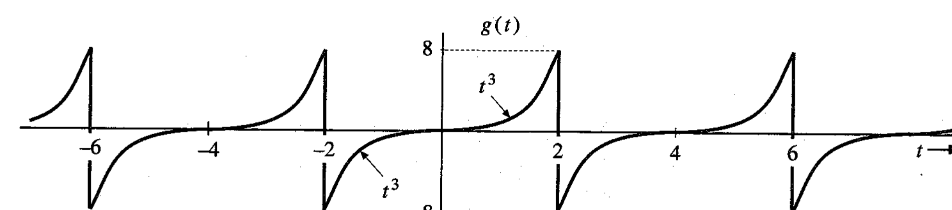


Figure P2.1-5

2.1-6 Find the power and the rms value for the signals in: **(a)** Fig. 2.21b; **(b)** Fig. 2.22a; **(c)** Fig. 2.23; **(d)** Fig. P2.8-4a; **(e)** Fig. P2.8-4c.

2.1-7 Show that the power of a signal $g(t)$ given by

$$g(t) = \sum_{k=m}^{n} D_k e^{j\omega_k t} \qquad \omega_i \neq \omega_k \text{ for all } i \neq k$$

is (Parseval's theorem)

$$P_g = \sum_{k=m}^{n} |D_k|^2$$

2.1-8 Determine the power and the rms value for each of the following signals:

(a) $10 \cos\left(100t + \frac{\pi}{3}\right)$ (b) $10 \cos\left(100t + \frac{\pi}{3}\right) + 16 \sin\left(150t + \frac{\pi}{5}\right)$

(c) $(10 + 2 \sin 3t) \cos 10t$ (d) $10 \cos 5t \cos 10t$

(e) $10 \sin 5t \cos 10t$ (f) $e^{j\alpha t} \cos \omega_0 t$

2.2-1 Show that an exponential e^{-at} starting at $-\infty$ is neither an energy nor a power signal for any real value of a. However, if a is imaginary, it is a power signal with power $P_g = 1$ regardless of the value of a.

2.3-1 In Fig. P2.3-1, the signal $g_1(t) = g(-t)$. Express signals $g_2(t)$, $g_3(t)$, $g_4(t)$, and $g_5(t)$ in terms of signals $g(t)$, $g_1(t)$, and their time-shifted, time-scaled, or time-inverted versions. For instance $g_2(t) = g(t - T) + g_1(t - T)$ for some suitable value of T. Similarly, both $g_3(t)$ and $g_4(t)$ can be expressed as $g(t - T) + g(t + T)$ for some suitable value of T. $g_5(t)$ can be expressed as $g(t)$ time-shifted, time-scaled, and then multiplied by a constant. (These operations may be performed in any order).

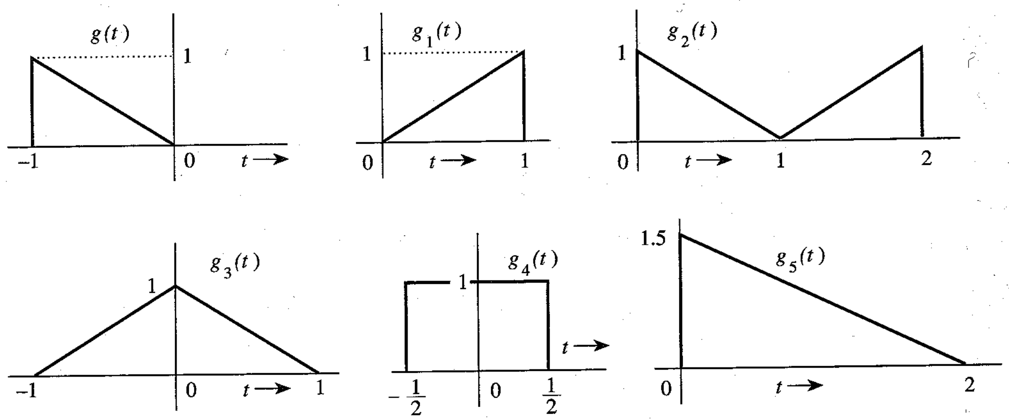


Figure P2.3-1

2.3-2 For the signal $g(t)$ shown in Fig. P2.3-2, sketch the signals: **(a)** $g(-t)$; **(b)** $g(t+6)$; **(c)** $g(3t)$; **(d)** $g(6-t)$.

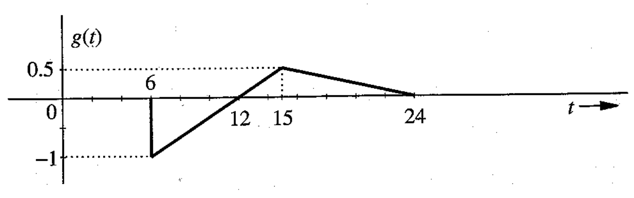


Figure P2.3-2

2.3-3 For the signal $g(t)$ shown in Fig. P2.3-3, sketch: **(a)** $g(t-4)$; **(b)** $g(t/1.5)$; **(c)** $g(2t-4)$ **(d)** $g(2-t)$. *Hint*: Recall that replacing t with $t - T$ delays the signal by T. Thus, $g(2t-4)$ is $g(2t)$ with t replaced by $t-2$. Similarly, $g(2-t)$ is $g(-t)$ with t replaced by $t-2$.

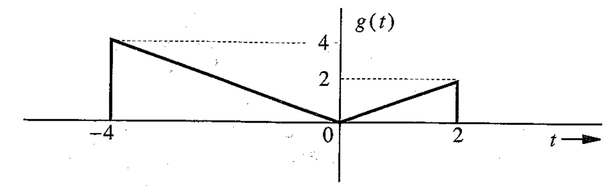


Figure P2.3-3

2.3-4 For an energy signal $g(t)$ with energy E_g, show that the energy of any one of the signals $-g(t)$, $g(-t)$, and $g(t-T)$ is E_g. Show also that the energy of $g(at)$ as well as $g(at-b)$ is E_g/a. This shows that time inversion and time shifting do not affect signal energy. On the other hand, time compression of a signal by a factor a reduces the energy by the factor a. What is the effect on signal energy if the signal is: **(a)** time-expanded by a factor a $(a > 1)$; **(b)** multiplied by a constant a?

2.4-1 Simplify the following expressions:

(a) $\left(\frac{\sin t}{t^2+2}\right)\delta(t)$ (b) $\left(\frac{j\omega+2}{\omega^2+9}\right)\delta(\omega)$

(c) $\left[e^{-t}\cos(3t - 60°)\right]\delta(t)$ (d) $\left[\frac{\sin \frac{\pi}{2}(t-2)}{t^2+4}\right]\delta(t-1)$

(e) $\left(\frac{1}{j\omega+2}\right)\delta(\omega+3)$ (f) $\left(\frac{\sin k\omega}{\omega}\right)\delta(\omega)$

Hint: Use Eq. (2.18). For part **(f)** use L'Hôpital's rule.

2.4-2 Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} g(\tau)\delta(t-\tau)\,d\tau$ (b) $\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau$

(c) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt$ (d) $\int_{-\infty}^{\infty} \delta(t-2)\sin \pi t\,dt$

(e) $\int_{-\infty}^{\infty} \delta(t+3)e^{-t}\,dt$ (f) $\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)\,dt$

(g) $\int_{-\infty}^{\infty} g(2-t)\delta(3-t)\,dt$ (h) $\int_{-\infty}^{\infty} e^{(x-1)}\cos\frac{\pi}{2}(x-5)\delta(x-3)\,dx$

Hint: $\delta(x)$ is located at $x = 0$. For example, $\delta(1-t)$ is located at $1 - t = 0$; that is, at $t = 1$, and so on.

2.4-3 Prove that

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Hence, show that

$$\delta(\omega) = \frac{1}{2\pi}\delta(f) \qquad \text{where} \quad \omega = 2\pi f$$

Hint: Show that

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|}\phi(0)$$

2.5-1 Derive Eq. (2.26) in an alternate way by observing that $\mathbf{e} = (\mathbf{g} - c\mathbf{x})$, and

$$|\mathbf{e}|^2 = (\mathbf{g}-c\mathbf{x})\cdot(\mathbf{g}-c\mathbf{x}) = |\mathbf{g}|^2 + c^2|\mathbf{x}|^2 - 2c\mathbf{g}\cdot\mathbf{x}$$

2.5-2 For the signals $g(t)$ and $x(t)$ shown in Fig. P2.5-2, find the component of the form $x(t)$ contained in $g(t)$. In other words, find the optimum value of c in the approximation $g(t) \approx cx(t)$ so that the error signal energy is minimum. What is the error signal energy?

Figure P2.5-2

2.5-3 For the signals $g(t)$ and $x(t)$ shown in Fig. P2.5-2, find the component of the form $g(t)$ contained in $x(t)$. In other words, find the optimum value of c in the approximation $x(t) \approx cg(t)$ so that the error signal energy is minimum. What is the error signal energy?

2.5-4 Repeat Prob. 2.5-2 if $x(t)$ is the sinusoid pulse shown in Fig. P2.5-4.

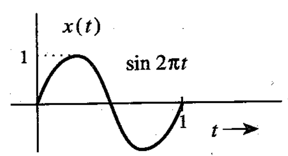


Figure P2.5-4

2.5-5 Energies of the two energy signals $x(t)$ and $y(t)$ are E_x and E_y, respectively.

(a) If $x(t)$ and $y(t)$ are orthogonal, then show that the energy of the signal $x(t) + y(t)$ is identical to the energy of the signal $x(t) - y(t)$, and is given by $E_x + E_y$.

(b) If $x(t)$ and $y(t)$ are orthogonal, find the energies of signals $c_1x(t) + c_2y(t)$ and $c_1x(t) - c_2y(t)$.

(c) We define E_{xy}, the cross energy of the two energy signals $x(t)$ and $y(t)$, as

$$E_{xy} = \int_{-\infty}^{\infty} x(t)y^*(t)\,dt$$

If $z(t) = x(t) \pm y(t)$, then show that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

2.5-6 Let $x_1(t)$ and $x_2(t)$ be two unit energy signals orthogonal over an interval from $t = t_1$ to t_2. We can represent $x_1(t)$ and $x_2(t)$ by two unit length, orthogonal vectors $(\mathbf{x}_1, \mathbf{x}_2)$. Consider a signal $g(t)$ where

$$g(t) = c_1x_1(t) + c_2x_2(t) \qquad t_1 \le t \le t_2$$

This signal can be represented as a vector $\mathbf{g}$ by a point (c_1, c_2) in the x_1–x_2 plane.

(a) Determine the vector representation of the following six signals in this two-dimensional vector space:

(i) $g_1(t) = 2x_1(t) - x_2(t)$ **(ii)** $g_2(t) = -x_1(t) + 2x_2(t)$
(iii) $g_3(t) = -x_2(t)$ **(iv)** $g_4(t) = x_1(t) + 2x_2(t)$
(v) $g_5(t) = 2x_1(t) + x_2(t)$ **(vi)** $g_6(t) = 3x_1(t)$

(b) Point out pairs of mutually orthogonal vectors among these six vectors. Verify that the pairs of signals corresponding to these orthogonal vectors are also orthogonal.

2.6-1 Find the correlation coefficient c_n of signal $x(t)$ and each of the four pulses $g_1(t)$, $g_2(t)$, $g_3(t)$, and $g_4(t)$ shown in Fig. P2.6-1. Which pair of pulses would you select for a binary communication in order to provide maximum margin against the noise along the transmission path?

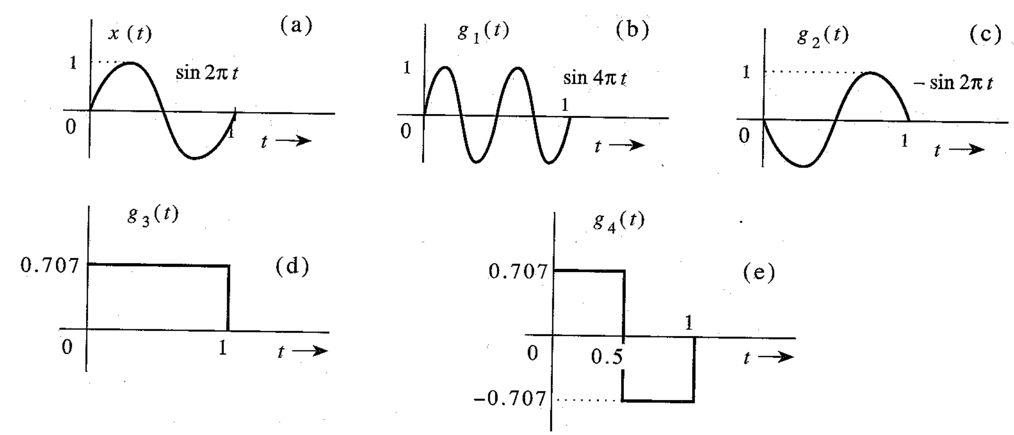


Figure P2.6-1

2.8-1 **(a)** Sketch the signal $g(t) = t^2$ and find the trigonometric Fourier series to represent $g(t)$ over the interval $(-1, 1)$. Sketch the Fourier series $\varphi(t)$ for all values of t.

(b) Verify Parseval's theorem [Eq. (2.90)] for this case, given that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

2.8-2 **(a)** Sketch the signal $g(t) = t$ and find the trigonometric Fourier series to represent $g(t)$ over the interval $(-\pi, \pi)$. Sketch the Fourier series $\varphi(t)$ for all values of t.

(b) Verify Parseval's theorem [Eq. (2.90)] for this case, given that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

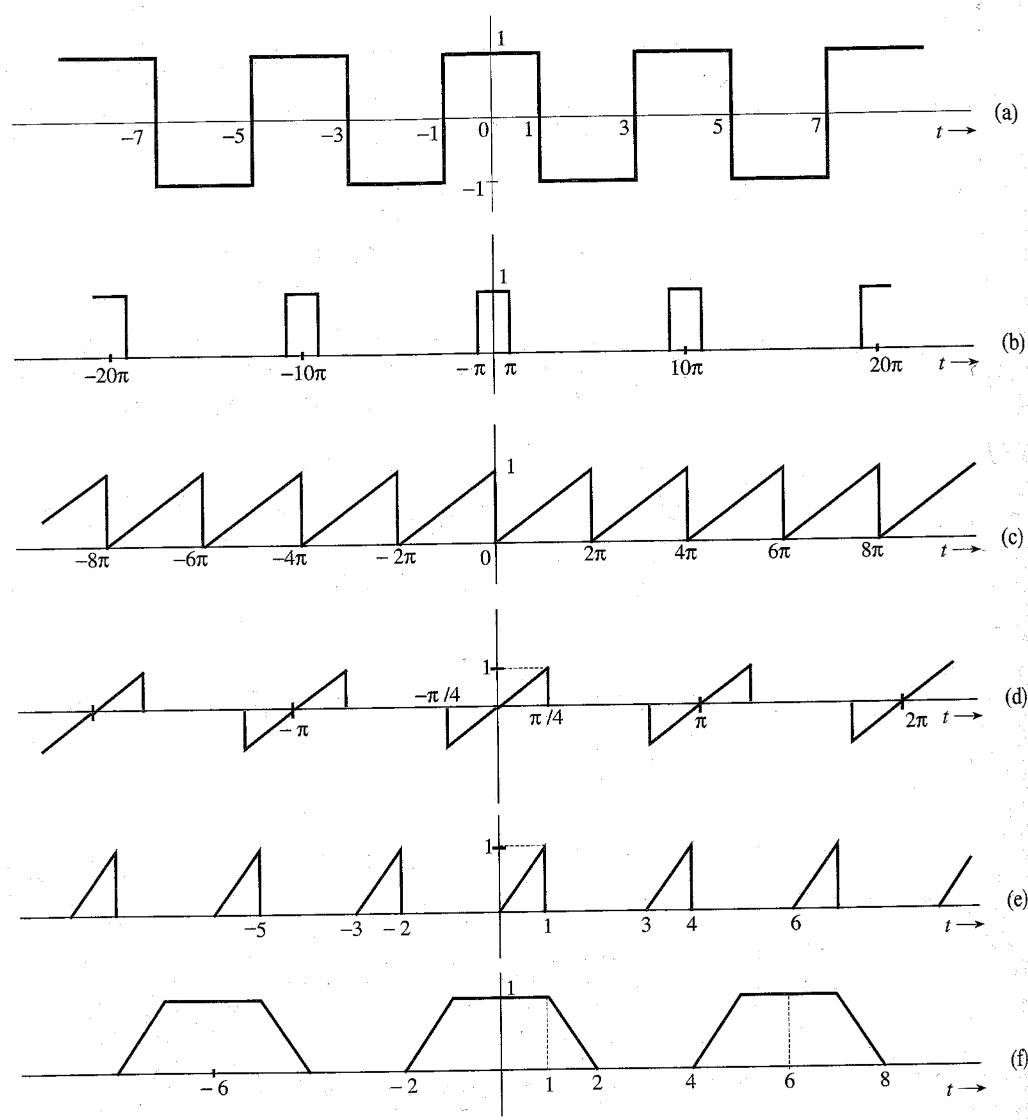


Figure P2.8-4

2.8-3 If a periodic signal satisfies certain symmetry conditions, the evaluation of the Fourier series components is somewhat simplified. Show that:

(a) If $g(t) = g(-t)$ (even symmetry), then all the sine terms in the Fourier series vanish ($b_n = 0$).

(b) If $g(t) = -g(-t)$ (odd symmetry), then the dc and all the cosine terms in the Fourier series vanish ($a_0 = a_n = 0$).

Further, show that in each case the Fourier coefficients can be evaluated by integrating the periodic signal over the half-cycle only. This is because the entire information of one cycle is implicit in a half-cycle due to symmetry. *Hint*: If $g_e(t)$ and $g_o(t)$ are even and odd functions, respectively, of t, then (assuming no impulse or its derivative at the origin)

$$\int_{-a}^{a} g_e(t)\,dt = 2\int_{0}^{a} g_e(t)\,dt \qquad \text{and} \qquad \int_{-a}^{a} g_o(t)\,dt = 0$$

Also the product of an even and an odd function is an odd function, the product of two odd functions is an even function, and the product of two even functions is an even function.

2.8-4 For each of the periodic signals shown in Fig. P2.8-4, find the compact trigonometric Fourier series and sketch the amplitude and phase spectra. If either the sine or the cosine terms are absent in the Fourier series, explain why.

2.8-5 **(a)** Show that an arbitrary function $g(t)$ can be expressed as a sum of an even function $g_e(t)$ and an odd function $g_o(t)$:

$$g(t) = g_e(t) + g_o(t)$$

Hint:

$$g(t) = \underbrace{\frac{1}{2}[g(t) + g(-t)]}_{g_e(t)} + \underbrace{\frac{1}{2}[g(t) - g(-t)]}_{g_o(t)}$$

(b) Determine the odd and even components of the functions: **(i)** $u(t)$; **(ii)** $e^{-at}u(t)$; **(iii)** e^{jt}.

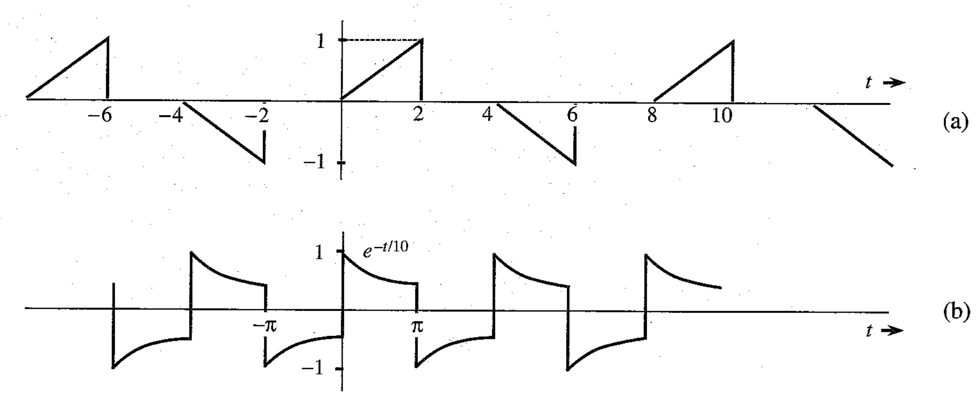


Figure P2.8-6

2.8-6 If the two halves of one period of a periodic signal are of identical shape except that one is the negative of the other, the periodic signal is said to have a **half-wave symmetry**. If a periodic signal $g(t)$ with a period T_0 satisfies the half-wave symmetry condition, then

$$g\left(t - \frac{T_0}{2}\right) = -g(t)$$

In this case, show that all the even-numbered harmonics vanish, and that the odd-numbered harmonic coefficients are given by

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} g(t)\cos n\omega_0 t\, dt \quad \text{and} \quad b_n = \frac{4}{T_0}\int_0^{T_0/2} g(t)\sin n\omega_0 t\, dt$$

Using these results, find the Fourier series for the periodic signals in Fig. P2.8-6.

2.9-1 For each of the periodic signals in Fig. P2.8-4, find exponential Fourier series and sketch the corresponding spectra.

2.9-2 A periodic signal $g(t)$ is expressed by the following Fourier series:

$$g(t) = 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t + \frac{2\pi}{3}\right)$$

(a) Sketch the amplitude and phase spectra for the trigonometric series.

(b) By inspection of spectra in part **(a)**, sketch the exponential Fourier series spectra.

(c) By inspection of spectra in part **(b)**, write the exponential Fourier series for $g(t)$.

2.9-3 Figure P2.9-3 shows the trigonometric Fourier spectra of a periodic signal $g(t)$.

(a) By inspection of Fig. P2.9-3, find the trigonometric Fourier series representing $g(t)$.

(b) By inspection of Fig. P2.9-3, sketch the exponential Fourier spectra of $g(t)$.

(c) By inspection of the exponential Fourier spectra obtained in part **(b)**, find the exponential Fourier series for $g(t)$.

(d) Show that the series found in parts **(a)** and **(c)** are equivalent.

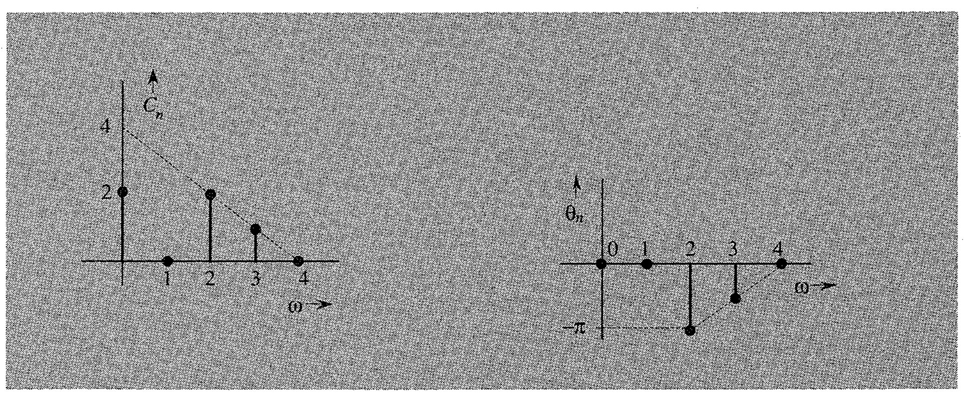


Figure P2.9-3

2.9-4 Show that the coefficients of the exponential Fourier series of an even periodic signal are real and those of an odd periodic signal are imaginary.

3 ANALYSIS AND TRANSMISSION OF SIGNALS

Electrical engineers instinctively think of signals in terms of their frequency spectra and think of systems in terms of their frequency responses. Even teenagers know about audio signals having a bandwidth of 20 kHz and good-quality loud speakers responding up to 20 kHz. This is basically thinking in the frequency domain. In the last chapter we discussed spectral representation of periodic signals (Fourier series). In this chapter we extend this spectral representation to aperiodic signals.

3.1 APERIODIC SIGNAL REPRESENTATION BY FOURIER INTEGRAL

Applying a limiting process, we now show that an aperiodic signal can be expressed as a continuous sum (integral) of everlasting exponentials. To represent an aperiodic signal $g(t)$, such as the one shown in Fig. 3.1a by everlasting exponential signals, let us construct a new periodic signal $g_{T_0}(t)$ formed by repeating the signal $g(t)$ every T_0 seconds, as shown in Fig. 3.1b. The period T_0 is made long enough to avoid overlap between the repeating pulses. The periodic signal $g_{T_0}(t)$ can be represented by an exponential Fourier series. If we let $T_0 \to \infty$, the pulses in the periodic signal repeat after an infinite interval, and therefore

$$\lim_{T_0\to\infty} g_{T_0}(t) = g(t)$$

Thus, the Fourier series representing $g_{T_0}(t)$ will also represent $g(t)$ in the limit $T_0 \to \infty$. The exponential Fourier series for $g_{T_0}(t)$ is given by

$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \tag{3.1}$$

in which

$$D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t) e^{-jn\omega_0 t}\, dt \tag{3.2a}$$

Chapter 2

2.1-1 Let us denote the signal in question by $g(t)$ and its energy by E_g. For parts (a) and (b)

$$E_g = \int_0^{2\pi} \sin^2 t\,dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt = \pi + 0 = \pi$$

$$\text{(c)} \qquad E_g = \int_{2\pi}^{4\pi} \sin^2 t\,dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\,dt = \pi + 0 = \pi$$

$$\text{(d)} \qquad E_g = \int_0^{2\pi} (2\sin t)^2\,dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-2 (a) $E_x = \int_0^2 (1)^2 dt = 2.$ $\qquad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4. \quad E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x\pm y} = E_x + E_y$.

$$\text{(b)}\,E_x = \int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt = 2\pi, \qquad E_y = \int_0^{\pi/2} (1)^2 dt + \int_{\pi/2}^{\pi} (-1)^2 dt + \int_{\pi}^{3\pi/2} (1)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 2\pi$$

$$E_{x+y} = \int_0^{\pi/2} (2)^2 dt + \int_{\pi/2}^{3\pi/2} (0)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 4\pi$$

Similarly, we can show that $E_{x-y} = 4\pi$ Therefore $E_{x\pm y} = E_x + E_y$. We are tempted to conclude that $E_{x\pm y} = E_x + E_y$ in general. Let us see.

$$\text{(c)} \qquad E_x = \int_0^{\pi/4} (1)^2 dt + \int_{\pi/4}^{\pi} (-1)^2 dt = \pi \qquad E_y = \int_0^{\pi} (1)^2 dt = \pi$$

$$E_{x+y} = \int_0^{\pi/4} (2)^2 dt + \int_{\pi/4}^{\pi} (0)^2 dt = \pi \qquad E_{x-y} = \int_0^{\pi/4} (0)^2 dt + \int_{\pi/4}^{\pi} (-2)^2 dt = 3\pi$$

Therefore, in general $E_{x\pm y} \neq E_x + E_y$

2.1-3

$$P_g = \frac{1}{T_0}\int_0^{T_0} C^2 \cos^2(\omega_0 t + \theta)\,dt = \frac{C^2}{2T_0}\int_0^{T_0} [1 + \cos(2\omega_0 t + 2\theta)]\,dt$$

$$= \frac{C^2}{2T_0}\left[\int_0^{T_0} dt + \int_0^{T_0} \cos(2\omega_0 t + 2\theta)\,dt\right] = \frac{C^2}{2T_0}[T_0 + 0] = \frac{C^2}{2}$$

2.1-4 This problem is identical to Example 2.2b, except that $\omega_1 \neq \omega_2$. In this case, the third integral in P_g (see p. 19 is not zero. This integral is given by

$$I_3 = \lim_{T\to\infty} \frac{2C_1C_2}{T}\int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2)\,dt$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}\left[\int_{-T/2}^{T/2} \cos(\theta_1 - \theta_2)\,dt + \int_{-T/2}^{T/2} \cos(2\omega_1 t + \theta_1 + \theta_2)\,dt\right]$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}[T\cos(\theta_1 - \theta_2)] + 0 = C_1C_2\cos(\theta_1 - \theta_2)$$

Therefore

$$P_y = \frac{C_1^2}{2} + \frac{C_2^2}{2} + C_1C_2\cos(\theta_1 - \theta_2)$$

2.1-5

$$P_g = \frac{1}{4}\int_{-2}^{2}(t^3)^2 dt = 64/7 \qquad \textbf{(a)}\ P_{-g} = \frac{1}{4}\int_{-2}^{2}(-t^3)^2 dt = 64/7$$

$$\textbf{(b)}\ P_{2y} = \frac{1}{4}\int_{-2}^{2}(2t^3)^2 dt = 4(64/7) = 256/7 \qquad \textbf{(c)}\ P_{cg} = \frac{1}{4}\int_{-2}^{2}(ct^3)^2 dt = 64c^2/7$$

Sign change of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor c^2.

2.1-6

$$\textbf{(a)}\quad P_g = \frac{1}{\pi}\int_0^{\pi}(e^{-t/2})^2\,dt = \frac{1}{\pi}\int_0^{\pi} e^{-t}\,dt = \frac{1}{\pi}[1 - e^{-\pi}]$$

$$\textbf{(b)}\quad P_y = \frac{1}{2\pi}\int_{-\pi}^{\pi} w^2(t)\,dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} dt = 0.5$$

$$\textbf{(c)}\quad P_g = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} w_0^2(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt = 1$$

$$\textbf{(d)}\quad P_y = \frac{1}{4}\int_{-2}^{2}(\pm 1)^2\,dt = 1$$

$$\textbf{(e)}\quad P_g = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{t}{2\pi}\right)^2 dt = \frac{1}{3}$$

2.1-7

$$P_y = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)g^*(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{r=m}^{n} D_k D_r^* e^{j(\omega_k - \omega_r)t}\,dt$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_y = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2\,dt = \sum_{k=m}^{n}|D_k|^2$$

2.1-8 **(a)** Power of a sinusoid of amplitude C is $C^2/2$ [Eq. (2.6a)] regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = (10)^2/2 = 50$.
(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq. (2.6b)]. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.
(c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence from Eq. (2.6b) $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.
(d) $10\cos 5t\cos 10t = 5(\cos 5t + \cos 15t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.
(e) $10\sin 5t\cos 10t = 5(\sin 15t - \sin 5t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.
(f) $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Using the result in Prob. 2.1-7, we obtain $P = (1/4) + (1/4) = 1/2$.

2.2-1 For a real a

$$E_g = \int_{-\infty}^{\infty}(e^{-at})^2\,dt = \int_{-\infty}^{\infty} e^{-2at}\,dt = \infty$$

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}(e^{-at})^2\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e^{-2at}\,dt = \infty$$

For imaginary a, let $a = jx$. Then

$$P_y = llim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}(e^{jxt})(e^{-jxt})\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} dt = 1$$

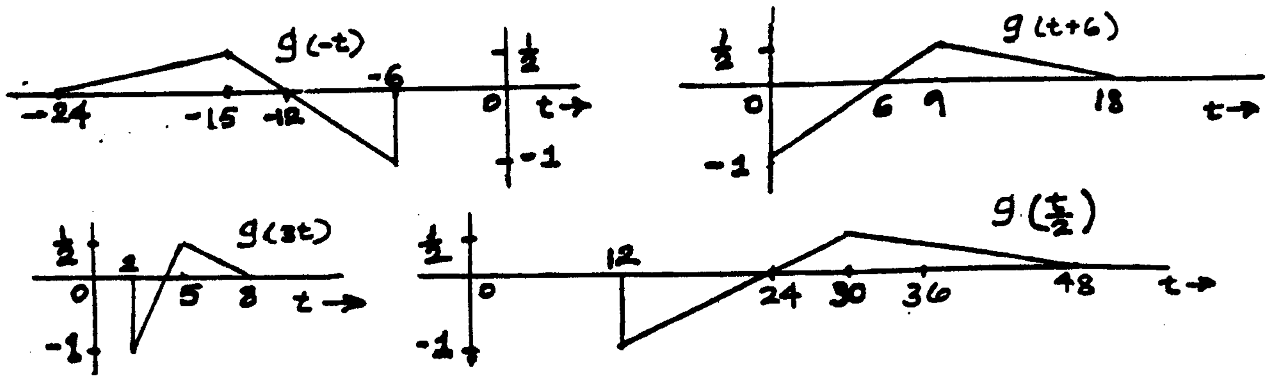


Fig. S2.3-2

Clearly, if a is real, e^{-at} is neither energy not power signal. However, if a is imaginary, it is a power signal with power 1.

2.3-1

$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiply with 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2} - 1)$.

2.3-2 All the signals are shown in Fig. S2.3-2.

2.3-3 All the signals are shown in Fig. S2.3-3

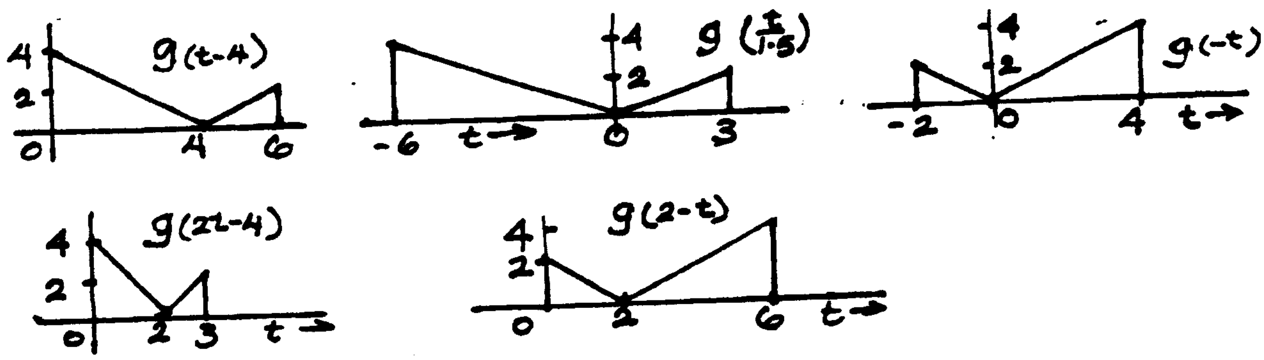


Fig. S2.3-3

2.3-4

$$E_{-g} = \int_{-\infty}^{\infty} [-g(t)]^2\, dt = \int_{-\infty}^{\infty} g^2(t)\, dt = E_g. \quad E_{g(-t)} = \int_{-\infty}^{\infty} [g(-t)]^2\, dt = \int_{-\infty}^{\infty} g^2(x)\, dx = E_g$$

$$E_{g(t-T)} = \int_{-\infty}^{\infty} [g(t-T)]^2\, dt = \int_{-\infty}^{\infty} g^2(x)\, dx = E_g. \quad E_{g(at)} = \int_{-\infty}^{\infty} [g(at)]^2\, dt = \frac{1}{a}\int_{-\infty}^{\infty} g^2(x)\, dx = E_g/a$$

$$E_{g(at-b)} = \int_{-\infty}^{\infty} [g(at-b)]^2\, dt = \frac{1}{a}\int_{-\infty}^{\infty} g^2(x)\, dx = E_g/a. \quad E_{g(t/a)} = \int_{-\infty}^{\infty} [g(t/a)]^2\, dt = a\int_{-\infty}^{\infty} g^2(x)\, dt = aE_g$$

$$E_{ag(t)} = \int_{-\infty}^{\infty} [ag(t)]^2\, dt = a^2\int_{-\infty}^{\infty} g^2(t)\, dt = a^2 E_g$$

2.4-1 Using the fact that $g(x)\delta(x) = g(0)\delta(x)$, we have

(a) 0 **(b)** $\frac{2}{9}\delta(\omega)$ **(c)** $\frac{1}{2}\delta(t)$ **(d)** $-\frac{1}{3}\delta(t-1)$ **(e)** $\frac{1}{2-j3}\delta(\omega+3)$ **(f)** $k\delta(\omega)$ (use L' Hôpital's rule)

2.4-2 In these problems remember that impulse $\delta(x)$ is located at $x = 0$. Thus, an impulse $\delta(t-\tau)$ is located at $\tau = t$, and so on.

(a) The impulse is located at $\tau = t$ and $g(\tau)$ at $\tau = t$ is $g(t)$. Therefore

$$\int_{-\infty}^{\infty} g(\tau)\delta(t-\tau)\,d\tau = g(t)$$

(b) The impulse $\delta(\tau)$ is at $\tau = 0$ and $g(t-\tau)$ at $\tau = 0$ is $g(t)$. Therefore

$$\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau = g(t)$$

Using similar arguments, we obtain
(c) 1 **(d)** 0 **(e)** e^3 **(f)** 5 **(g)** $g(-1)$ **(h)** $-e^2$

2.4-3 Letting $at = x$, we obtain (for $a > 0$)

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{a}\int_{-\infty}^{\infty} \phi(\frac{x}{a})\delta(x)\,dx = \frac{1}{a}\phi(0)$$

Similarly for $a < 0$, we show that this integral is $-\frac{1}{a}\phi(0)$. Therefore

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|}\phi(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt$$

Therefore

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

2.5-1 Trivial. Take the derivative of $|\mathbf{e}|^2$ with respect to c and equate it to zero.

2.5-2 **(a)** In this case $E_x = \int_0^1 dt = 1$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\,dt = \frac{1}{1}\int_0^1 t\,dt = 0.5$$

(b) Thus, $g(t) \approx 0.5x(t)$, and the error $e(t) = t - 0.5$ over $(0 \le t \le 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 (t-0.5)^2\,dt = 1/12$$

The error $(t - 0.5)$ is orthogonal to $x(t)$ because

$$\int_0^1 (t-0.5)(1)\,dt = 0$$

Note that $E_g = c^2E_x + E_e$. To explain these results in terms of vector concepts we observe from Fig. 2.15 that the error vector $\mathbf{e}$ is orthogonal to the component $c\mathbf{x}$. Because of this orthogonality, the length-square of $\mathbf{g}$ [energy of $g(t)$] is equal to the sum of the square of the lengths of $c\mathbf{x}$ and $\mathbf{e}$ [sum of the energies of $cx(t)$ and $e(t)$].

2.5-3 In this case $E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3$, and

$$c = \frac{1}{E_g}\int_0^1 x(t)g(t)\,dt = 3\int_0^1 t\,dt = 1.5$$

Thus, $x(t) \approx 1.5g(t)$, and the error $e(t) = x(t) - 1.5g(t) = 1 - 1.5t$ over $(0 \le t \le 1)$, and zero outside this interval. Also E_e (the energy of the error) is $E_e = \int_0^1 (1-1.5t)^2\,dt = 1/4$.

2.5-4 **(a)** In this case $E_x = \int_0^1 \sin^2 2\pi t\,dt = 0.5$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\,dt = \frac{1}{0.5}\int_0^1 t\sin 2\pi t\,dt = -1/\pi$$

(b) Thus, $g(t) \approx -(1/\pi)x(t)$, and the error $e(t) = t + (1/\pi)\sin 2\pi t$ over $(0 \le t \le 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 [t - (1/\pi)\sin 2\pi t]^2\,dt = \frac{1}{3} - \frac{1}{2\pi^2}$$

The error $[t + (1/\pi)\sin 2\pi t]$ is orthogonal to $x(t)$ because

$$\int_0^1 \sin 2\pi t[t + (1/\pi)\sin 2\pi t]\,dt = 0$$

Note that $E_g = c^2 E_x + E_e$. To explain these results in terms of vector concepts we observe from Fig. 2.15 that the error vector **e** is orthogonal to the component c**x**. Because of this orthogonality, the length of **f** [energy of $g(t)$] is equal to the sum of the square of the lengths of c**x** and **e** [sum of the energies of $cx(t)$ and $e(t)$].

2.5-5 **(a)** If $x(t)$ and $y(t)$ are orthogonal, then we can show the energy of $x(t) \pm y(t)$ is $E_x + E_y$.

$$\int_{-\infty}^{\infty} |x(t) \pm y(t)|^2\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt \pm \int_{-\infty}^{\infty} x(t)y^*(t)\,dt \pm \int_{-\infty}^{\infty} x^*(t)y(t)\,dt \qquad (1)$$

$$= \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt \qquad (2)$$

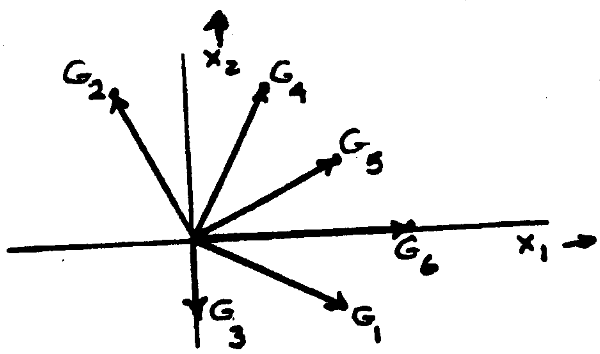
The last result follows from the fact that because of orthogonality, the two integrals of the cross products $x(t)y^*(t)$ and $x^*(t)y(t)$ are zero [see Eq. (2.40)]. Thus the energy of $x(t) + y(t)$ is equal to that of $x(t) - y(t)$ if $x(t)$ and $y(t)$ are orthogonal.

(b) Using similar argument, we can show that the energy of $c_1x(t) + c_2y(t)$ is equal to that of $c_1x(t) - c_2y(t)$ if $x(t)$ and $y(t)$ are orthogonal. This energy is given by $|c_1|^2E_x + |c_2|^2E_y$.

(c) If $z(t) = x(t) \pm y(t)$, then it follows from Eq. (1) in the above derivation that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

2.5-6 $\mathbf{g}_1(2, -1)$, $\mathbf{g}_2(-1, 2)$, $\mathbf{g}_3(0, -2)$, $\mathbf{g}_4(1, 2)$, $\mathbf{g}_5(2, 1)$, and $\mathbf{g}_6(3, 0)$. From Fig. S2.5-6, we see that pairs $(\mathbf{g}_3, \mathbf{g}_6)$, $(\mathbf{g}_1, \mathbf{g}_4)$ and $(\mathbf{g}_2, \mathbf{g}_5)$ are orthogonal. We can verify this also analytically.



gig. S2.5-6

$$\mathbf{g}_3 \cdot \mathbf{g}_6 = (0 \times 3) + (-2 \times 0) = 0$$
$$\mathbf{g}_1 \cdot \mathbf{g}_4 = (2 \times 1) + (-1 \times 2) = 0$$
$$\mathbf{g}_2 \cdot \mathbf{g}_5 = (-1 \times 2) + (2 \times 1) = 0$$

We can show that the corresponding signal pairs are also orthogonal.

$$\int_{-\infty}^{\infty} g_3(t)g_6(t)\,dt = \int_{-\infty}^{\infty} [-x_2(t)][3x_1(t)]\,dt = 0$$

$$\int_{-\infty}^{\infty} g_1(t)g_4(t)\,dt = \int_{-\infty}^{\infty} [2x_1(t) - x_2(t)][x_1(t) + 2x_2(t)]\,dt = 0$$

$$\int_{-\infty}^{\infty} g_2(t)g_5(t)\,dt = \int_{-\infty}^{\infty} [-x_1(t) + 2x_2(t)][2x_1(t) + x_2(t)]\,dt = 0$$

In deriving these results, we used the fact that $\int_{-\infty}^{\infty} x_1^2 dt = \int_{-\infty}^{\infty} x_2^2(t) dt = 1$ and $\int_{-\infty}^{\infty} x_1(t)x_2(t)\, dt = 0$

2.6-1 We shall compute c_n using Eq. (2.48) for each of the 4 cases. Let us first compute the energies of all the signals.

$$E_x = \int_0^1 \sin^2 2\pi t\, dt = 0.5$$

In the same way we find $E_{g_1} = E_{g_2} = E_{g_3} = E_{g_4} = 0.5$.
Using Eq. (2.48), the correlation coefficients for four cases are found as

(1) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 \sin 2\pi t \sin 4\pi t\, dt = 0$ (2) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 (\sin 2\pi t)(-\sin 2\pi t)\, dt = -1$

(3) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 0.707 \sin 2\pi t\, dt = 0$ (4) $\frac{1}{\sqrt{(0.5)(0.5)}} \left[\int_0^{0.5} 0.707 \sin 2\pi t\, dt - \int_{0.5}^1 0.707 \sin 2\pi t\, dt\right] = 1.414/\pi$

Signals $x(t)$ and $g_2(t)$ provide the maximum protection against noise.

2.8-1 Here $T_0 = 2$, so that $\omega_0 = 2\pi/2 = \pi$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \sin n\pi t \qquad -1 \le t \le 1$$

where

$$a_0 = \frac{1}{2}\int_{-1}^{1} t^2\, dt = \frac{1}{3}, \qquad a_n = \frac{2}{2}\int_{-1}^{1} t^2 \cos n\pi t\, dt = \frac{4(-1)^n}{\pi^2 n^2}, \qquad b_n = \frac{2}{2}\int_{-1}^{1} t^2 \sin n\pi t\, dt = 0$$

Therefore

$$g(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi t \qquad -1 \le t \le 1$$

Figure S2.8-1 shows $g(t) = t^2$ for all t and the corresponding Fourier series representing $g(t)$ over $(-1, 1)$.

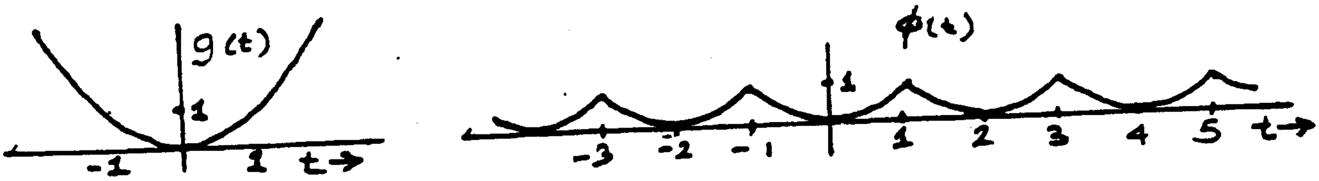


Fig. S2.8-1

The power of $g(t)$ is

$$P_g = \frac{1}{2}\int_{-1}^{1} t^4\, dt = \frac{1}{5}$$

Moreover, from Parseval's theorem [Eq. (2.90)]

$$P_g = C_0^2 + \sum_{1}^{\infty} \frac{C_n^2}{2} = \left(\frac{1}{3}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{\pi^2 n^2}\right)^2 = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{1}{9} + \frac{8}{90} = \frac{1}{5}$$

(b) If the N-term Fourier series is denoted by $x(t)$, then

$$x(t) = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{N-1} \frac{(-1)^n}{n^2} \cos n\pi t \qquad -1 \le t \le 1$$

The power P_x is required to be $99\% P_g = 0.198$. Therefore

$$P_x = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{N-1}\frac{1}{n^4} = 0.198$$

For $N = 1$, $P_x = 0.1111$; for $N = 2$, $P_x = 0.19323$. For $N = 3$, $P_x = 0.19837$, which is greater than 0.198. Thus, $N = 3$.

2.8-2 Here $T_0 = 2\pi$, so that $\omega_0 = 2\pi/2\pi = 1$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \qquad -\pi \le t \le \pi$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\,dt = 0, \qquad a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\cos nt\,dt = 0, \qquad b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\sin nt\,dt = \frac{2(-1)^{n+1}}{n}$$

Therefore

$$g(t) = 2(-1)^{n+1}\sum_{n=1}^{\infty}\frac{1}{n}\sin nt \qquad -\pi \le t \le \pi$$

Figure S2.8-2 shows $g(t) = t$ for all t and the corresponding Fourier series to represent $g(t)$ over $(-\pi, \pi)$.

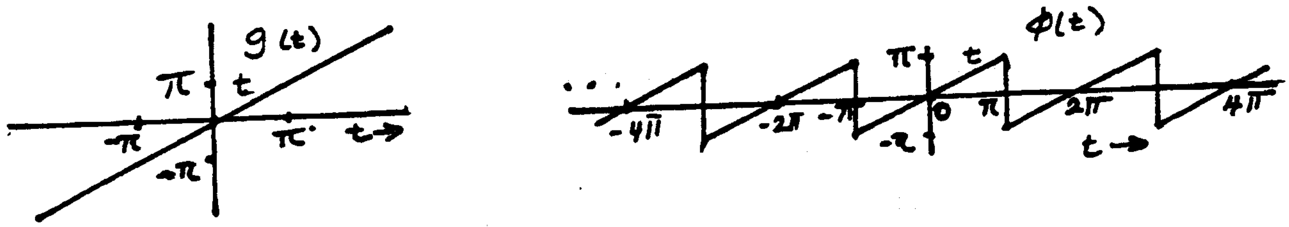


Fig. S2.8-2

The power of $g(t)$ is

$$P_g = \frac{1}{2\pi}\int_{-\pi}^{\pi} (t)^2\,dt = \frac{\pi^2}{3}$$

Moreover, from Parseval's theorem [Eq. (2.90)]

$$P_g = C_0^2 + \sum_{1}^{\infty}\frac{C_n^2}{2} = \frac{1}{2}\sum_{1}^{\infty}\frac{4}{n^2} = 2\sum_{1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{3}$$

(b) If the N-term Fourier series is denoted by $x(t)$, then

$$x(t) = 2(-1)^{n+1}\sum_{n=1}^{N}\frac{1}{n}\sin n\pi t \qquad -\pi \le t \le \pi$$

The power P_x is required to be $0.90 \times \frac{\pi^2}{3} = 0.3\pi^2$. Therefore

$$P_x = \frac{1}{2}\sum_{n=1}^{N}\frac{4}{n^2} = 0.3\pi^2$$

For $N = 1$, $P_x = 2$; for $N = 2$, $P_x = 2.5$, for $N = 5$, $P_x = 2.927$, which is less than $0.3\pi^2$. For $N = 6$, $P_x = 2.9825$, which is greater than $0.3\pi^2$. Thus, $N = 6$.

2.8-3 Recall that

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\,dt \tag{1a}$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\cos n\omega_0 t\,dt \tag{1b}$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\sin n\omega_0 t\,dt \tag{1c}$$

Recall also that $\cos n\omega_0 t$ is an even function and $\sin n\omega_0 t$ is an odd function of t. If $g(t)$ is an even function of t, then $g(t)\cos n\omega_0 t$ is also an even function and $g(t)\sin n\omega_0 t$ is an odd function of t. Therefore (see hint)

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} g(t)\,dt \tag{2a}$$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} g(t)\cos n\omega_0 t\,dt \tag{2b}$$

$$b_n = 0 \tag{2c}$$

Similarly, if $g(t)$ is an odd function of t, then $g(t)\cos n\omega_0 t$ is an odd function of t and $g(t)\sin n\omega_0 t$ is an even function of t. Therefore

$$a_0 = a_n = 0 \tag{3a}$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} g(t)\sin n\omega_0 t\,dt \tag{3b}$$

Observe that, because of symmetry, the integration required to compute the coefficients need be performed over only half the period.

2.8-4 (a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2}t\right)$$

$$a_0 = 0 \text{ (by inspection)}$$

$$a_n = \frac{4}{4}\left[\int_0^1 \cos\left(\frac{n\pi}{2}t\right)dt - \int_1^2 \cos\left(\frac{n\pi}{2}t\right)dt\right] = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

Therefore, the Fourier series for $g(t)$ is

$$g(t) = \frac{4}{\pi}\left(\cos\frac{\pi t}{2} - \frac{1}{3}\cos\frac{3\pi t}{2} + \frac{1}{5}\cos\frac{5\pi t}{2} - \frac{1}{7}\cos\frac{7\pi t}{2} + \cdots\right)$$

Here $b_n = 0$, and we allow C_n to take negative values. Figure S2.8-4a shows the plot of C_n.
(b) $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right)$$

$$a_0 = \frac{1}{5} \qquad \text{(by inspection)}$$

$$a_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right)dt = \frac{1}{5\pi}\left(\frac{5}{n}\right)\sin\left(\frac{n}{5}t\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

$$b_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right)dt = 0 \qquad \text{(integrand is an odd function of } t\text{)}$$

Here $b_n = 0$, and we allow C_n to take negative values. Note that $C_n = a_n$ for $n = 0, 1, 2, 3, \cdots$. Figure S2.8-4b shows the plot of C_n.
(c) $T_0 = 2\pi$, $\omega_0 = 1$.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \quad \text{with} \quad a_0 = 0.5 \quad \text{(by inspection)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos nt\,dt = 0, \qquad b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\sin nt\,dt = -\frac{1}{\pi n}$$

and

$$g(t) = 0.5 - \frac{1}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \frac{1}{4}\sin 4t + \cdots\right)$$
$$= 0.5 + \frac{1}{\pi}\left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t + \frac{\pi}{2}\right) + \cdots\right]$$

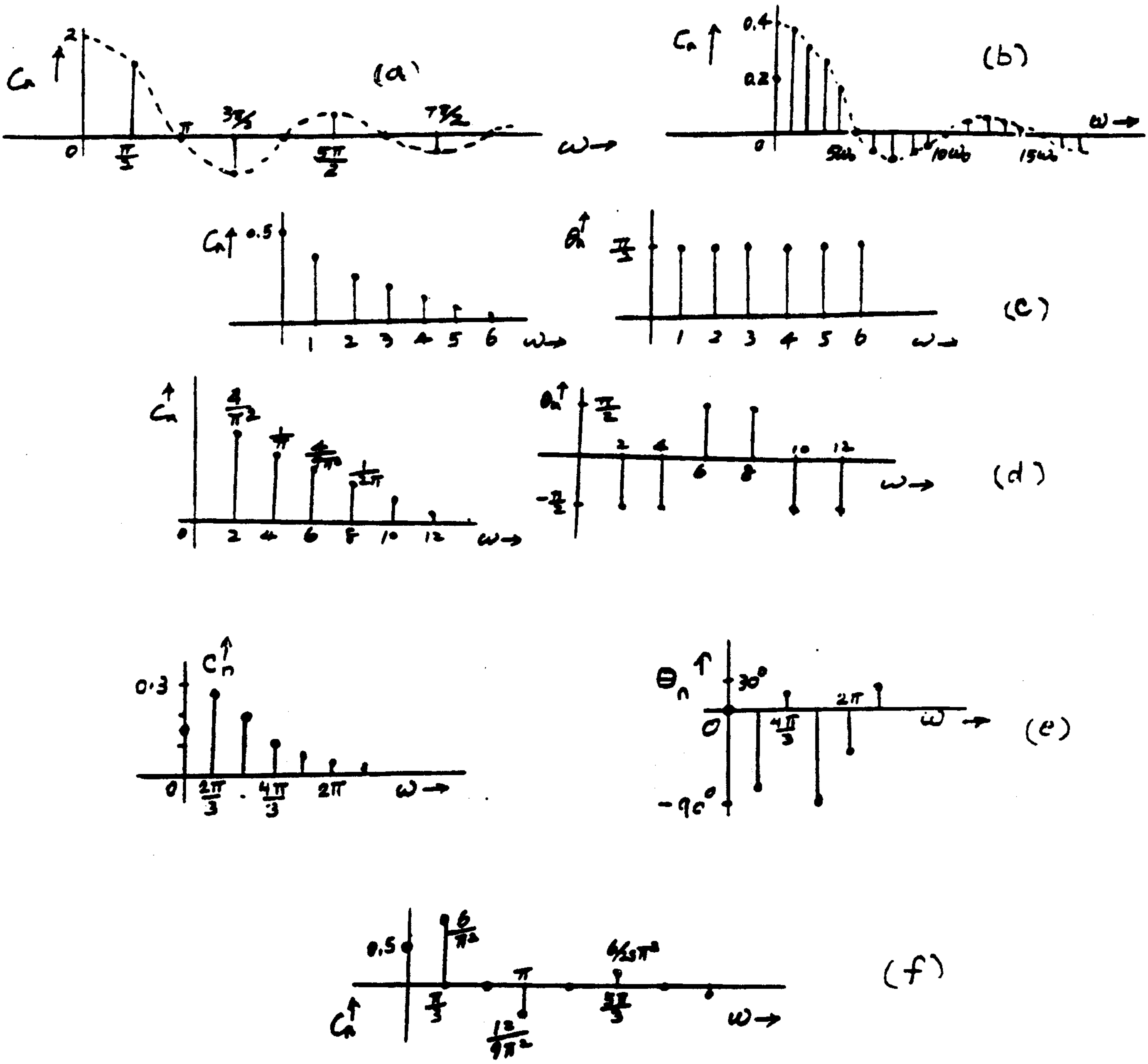


Fig. S2.8-4

The reason for vanishing of the cosines terms is that when 0.5 (the dc component) is subtracted from $g(t)$, the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-4c shows the plot of C_n and θ_n.

(d) $T_0 = \pi$, $\omega_0 = 2$ and $g(t) = \frac{4}{\pi}t$.

$a_0 = 0$ (by inspection).

$a_n = 0 \quad (n > 0)$ because of odd symmetry.

$$b_n = \frac{4}{\pi}\int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt\, dt = \frac{2}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

$$g(t) = \frac{4}{\pi^2}\sin 2t + \frac{1}{\pi}\sin 4t - \frac{4}{9\pi^2}\sin 6t - \frac{1}{2\pi}\sin 8t + \cdots$$

$$= \frac{4}{\pi^2}\cos\left(2t - \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(4t - \frac{\pi}{2}\right) + \frac{4}{9\pi^2}\cos\left(6t + \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(8t + \frac{\pi}{2}\right) + \cdots$$

Figure S2.8-4d shows the plot of C_n and θ_n.
(e) $T_0 = 3$, $\omega_0 = 2\pi/3$.

$$a_0 = \frac{1}{3}\int_0^1 t\,dt = \frac{1}{6}$$

$$a_n = \frac{2}{3}\int_0^1 t\cos\frac{2n\pi}{3}t\,dt = \frac{3}{2\pi^2 n^2}[\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1]$$

$$b_n = \frac{2}{3}\int_0^1 t\sin\frac{2n\pi}{3}t\,dt = \frac{3}{2\pi^2 n^2}[\sin\frac{2\pi n}{3} - \frac{2\pi n}{3}\cos\frac{2\pi n}{3}]$$

Therefore $C_0 = \frac{1}{6}$ and

$$C_n = \frac{3}{2\pi^2 n^2}\left[\sqrt{2 + \frac{4\pi^2 n^2}{9} - 2\cos\frac{2\pi n}{3} - \frac{4\pi n}{3}\sin\frac{2\pi n}{3}}\right] \quad \text{and} \quad \theta_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

(f) $T_0 = 6$, $\omega_0 = \pi/3$, $a_0 = 0.5$ (by inspection). Even symmetry; $b_n = 0$.

$$\begin{aligned} a_n &= \frac{4}{6}\int_0^3 g(t)\cos\frac{n\pi}{3}\,dt \\ &= \frac{2}{3}\left[\int_0^1 \cos\frac{n\pi}{3}\,dt + \int_1^2 (2-t)\cos\frac{n\pi}{3}t\,dt\right] \\ &= \frac{6}{\pi^2 n^2}\left[\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3}\right] \end{aligned}$$

$$g(t) = 0.5 + \frac{6}{\pi^2}\left(\cos\frac{\pi}{3}t - \frac{2}{9}\cos\pi t + \frac{1}{25}\cos\frac{5\pi}{3}t + \frac{1}{49}\cos\frac{7\pi}{3}t + \cdots\right)$$

Observe that even harmonics vanish. The reason is that if the dc (0.5) is subtracted from $g(t)$, the resulting function has half-wave symmetry. (See Prob. 2.8-6). Figure S2.8-4f shows the plot of C_n.

2.8-5

An even function $g_e(t)$ and an odd function $g_o(t)$ have the property that

$$g_e(t) = g_e(-t) \qquad \text{and} \qquad g_o(t) = -g_o(-t) \tag{1}$$

Every signal $g(t)$ can be expressed as a sum of even and odd components because

$$g(t) = \underbrace{\tfrac{1}{2}[g(t) + g(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[g(t) - g(-t)]}_{\text{odd}}$$

From the definitions in Eq. (1), it can be seen that the first component on the right-hand side is an even function, while the second component is odd. This is readily seen from the fact that replacing t by $-t$ in the first component yields the same function. The same maneuver in the second component yields the negative of that component.
To find the odd and the even components of $g(t) = u(t)$, we have

$$g(t) = g_e(t) + g_o(t)$$

where [from Eq. (1)]

$$g_e(t) = \tfrac{1}{2}[u(t) + u(-t)] = \frac{1}{2}$$

and

$$g_o(t) = \tfrac{1}{2}[u(t) - u(-t)] = \frac{1}{2}\text{sgn}(t)$$

The even and odd components of the signal $u(t)$ are shown in Fig. S2.8-5a.
Similarly, to find the odd and the even components of $g(t) = e^{-at}u(t)$, we have

$$g(t) = g_e(t) + g_o(t)$$

where

$$g_e(t) = \tfrac{1}{2}\left[e^{-at}u(t) + e^{at}u(-t)\right]$$

and

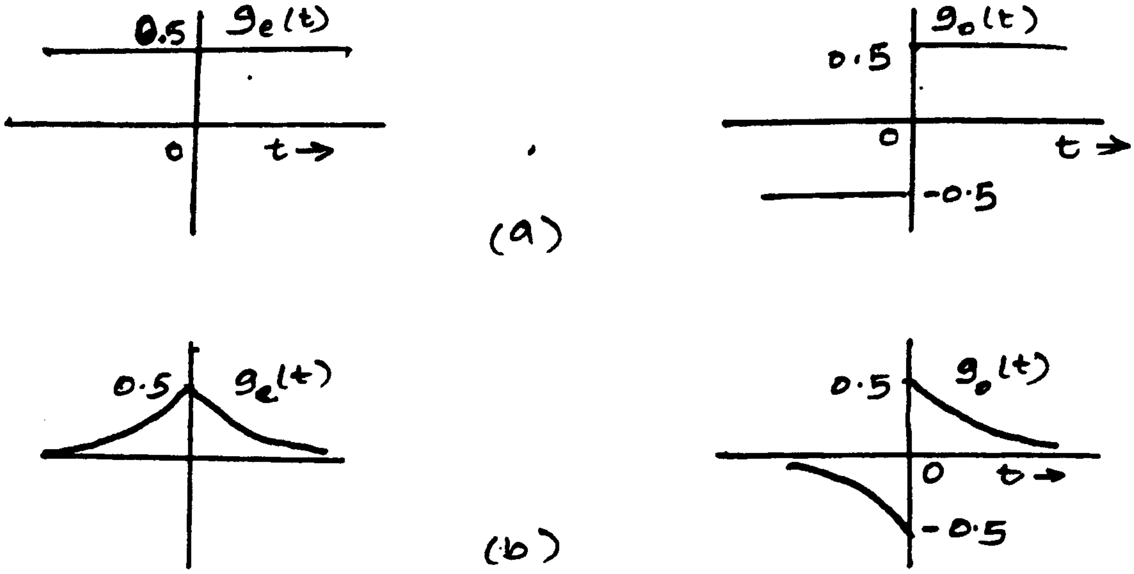


Fig. S2.8-5

$$g_o(t) = \frac{1}{2}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

The even and odd components of the signal $e^{-at}u(t)$ are shown in Fig. S2.8-5b.
For $g(t) = e^{jt}$, we have

$$e^{jt} = g_e(t) + g_o(t)$$

where

$$g_e(t) = \frac{1}{2}\left[e^{jt} + e^{-jt}\right] = \cos t$$

and

$$g_o(t) = \frac{1}{2}\left[e^{jt} - e^{-jt}\right] = j\sin t$$

2.8-6 **(a)** For half wave symmetry

$$g(t) = -g\left(t \pm \frac{T_0}{2}\right)$$

and

and $$a_n = \frac{2}{T_0}\int_0^{T_0} g(t)\cos n\omega_0 t\, dt = \frac{2}{T_0}\int_0^{T_0/2} g(t)\cos n\omega_0 t\, dt + \int_{T_0/2}^{T_0} g(t)\cos n\omega_0 t\, dt$$

Let $x = t - T_0/2$ in the second integral. This gives

$$\begin{aligned} a_n &= \frac{2}{T_0}\left[\int_0^{T_0/2} g(t)\cos n\omega_0 t\, dt + \int_0^{T_0/2} g\left(x + \frac{T_0}{2}\right)\cos n\omega_0\left(x + \frac{T_0}{2}\right) dx\right] \\ &= \frac{2}{T_0}\left[\int_0^{T_0/2} g(t)\cos n\omega_0 t\, dt + \int_0^{T_0/2} -g(x)[-\cos n\omega_0 x]\, dx\right] \\ &= \frac{4}{T_0}\left[\int_0^{T_0/2} g(t)\cos n\omega_0 t\, dt\right] \end{aligned}$$

In a similar way we can show that

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} g(t)\sin n\omega_0 t\, dt$$

(b) **(i)** $T_0 = 8$, $\omega_0 = \frac{\pi}{4}$, $a_0 = 0$ (by inspection). Half wave symmetry. Hence

$$a_n = \frac{4}{8}\left[\int_0^4 g(t)\cos\frac{n\pi}{4}t\,dt\right] = \frac{1}{2}\left[\int_0^2 \frac{t}{2}\cos\frac{n\pi}{4}t\,dt\right]$$

$$= \frac{4}{n^2\pi^2}\left(\cos\frac{n\pi}{2} + \frac{n\pi}{2}\sin\frac{n\pi}{2} - 1\right) \quad (n \text{ odd})$$

$$= \frac{4}{n^2\pi^2}\left(\frac{n\pi}{2}\sin\frac{n\pi}{2} - 1\right) \quad (n \text{ odd})$$

Therefore

$$a_n = \begin{cases} \frac{4}{n^2\pi^2}\left(\frac{n\pi}{2} - 1\right) & n = 1, 5, 9, 13, \cdots \\ -\frac{4}{n^2\pi^2}\left(\frac{n\pi}{2} + 1\right) & n = 3, 7, 11, 15, \cdots \end{cases}$$

Similarly

$$b_n = \frac{1}{2}\int_0^2 \frac{t}{2}\sin\frac{n\pi}{4}t\,dt = \frac{4}{n^2\pi^2}\left(\sin\frac{n\pi}{2} - \frac{n\pi}{2}\cos\frac{n\pi}{2}\right) = \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) \quad (n \text{ odd})$$

and

$$g(t) = \sum_{n=1,3,5,\cdots}^{\infty} a_n \cos\frac{n\pi}{4}t + b_n \sin\frac{n\pi}{4}t$$

(ii) $T_0 = 2\pi$, $\omega_0 = 1$, $a_0 = 0$ (by inspection). Half wave symmetry. Hence

$$g(t) = \sum_{n=1,3,5,\cdots}^{\infty} a_n \cos nt + b_n \sin nt$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/10}\cos nt\,dt$$

$$= \frac{2}{\pi}\left[\frac{e^{-t/10}}{n^2 + 0.01}(-0.1\cos nt + n\sin nt)\right]_0^{\pi} \quad (n \quad \text{odd})$$

$$= \frac{2}{\pi}\left[\frac{e^{-\pi/10}}{n^2 + 0.01}(0.1) - \frac{1}{n^2 + 0.01}(-0.1)\right]$$

$$= \frac{2}{10\pi(n^2 + 0.01)}(e^{-\pi/10} - 1) = \frac{0.0465}{n^2 + 0.01}$$

and

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/10}\sin nt\,dt$$

$$= \frac{2}{\pi}\left[\frac{e^{-t/10}}{n^2 + 0.01}(-0.1\sin nt - n\cos nt)\right]_0^{\pi} \quad (n \text{ odd})$$

$$= \frac{2n}{(n^2 + 0.01)}(e^{-\pi/10} - 1) = \frac{1.461n}{n^2 + 0.01}$$

2.9-1 **(a)**: $T_0 = 4$, $\omega_0 = \pi/2$. Also $D_0 = 0$ (by inspection).

$$D_n = \frac{1}{2\pi}\int_{-1}^{1} e^{-j(n\pi/2)t}\,dt - \int_1^3 e^{-j(n\pi/2)t}\,dt = \frac{2}{\pi n}\sin\frac{n\pi}{2} \qquad |n| \geq 1$$

(b) $T_0 = 10\pi$, $\omega_0 = 2\pi/10\pi = 1/5$

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t}, \qquad \text{where} \qquad D_n = \frac{1}{10\pi}\int_{\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt = \frac{j}{2\pi n}\left(-2j\sin\frac{n\pi}{5}\right) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

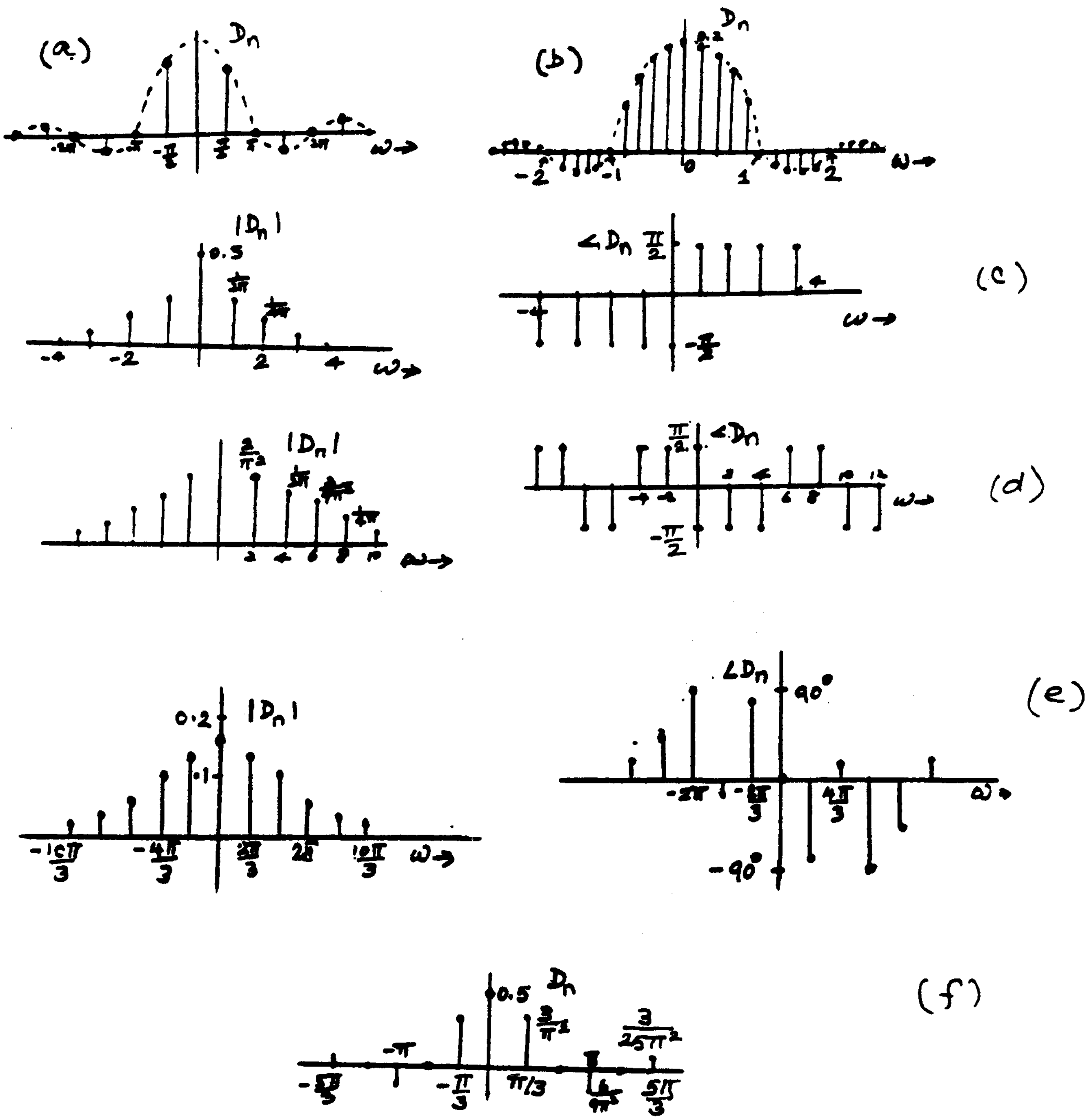


Fig. S2.9-1

(c)

$$g(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt}, \quad \text{where, by inspection} \quad D_0 = 0.5$$

$$D_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi} e^{-jnt}\,dt = \frac{j}{2\pi n}, \quad \text{so that} \quad |D_n| = \frac{1}{2\pi n}, \quad \text{and} \quad \angle D_n = \begin{cases} \frac{\pi}{2} & n > 0 \\ \frac{-\pi}{2} & n < 0 \end{cases}$$

(d) $T_0 = \pi$, $\omega_0 = 2$ and $D_n = 0$

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt}, \quad \text{where} \quad D_n = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j2nt}\,dt = \frac{-j}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

(e) $T_0 = 3, \omega_0 = \frac{2\pi}{3}$.

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{2\pi n}{3}t}, \qquad \text{where} \qquad D_n = \frac{1}{3}\int_0^1 t\,e^{-j\frac{2\pi n}{3}t}\,dt = \frac{3}{4\pi^2 n^2}\left[e^{-j\frac{2\pi n}{3}}\left(\frac{j2\pi n}{3}+1\right)-1\right]$$

Therefore

$$|D_n| = \frac{3}{4\pi^2 n^2}\left[\sqrt{2+\frac{4\pi^2 n^2}{9} - 2\cos\frac{2\pi n}{3} - \frac{4\pi n}{3}\sin\frac{2\pi n}{3}}\right] \quad \text{and} \quad \angle D_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

(f) $T_0 = 6$. $\omega_0 = \pi/3$ $D_0 = 0.5$

$$g(t) = 0.5 + \sum_{n=-\infty}^{\infty} D_n e^{\frac{j\pi n t}{3}}$$

$$D_n = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-\frac{j\pi n t}{3}}\,dt + \int_{-1}^{1} e^{-\frac{j\pi n t}{3}}\,dt + \int_1^2 (-t+2)e^{-\frac{j\pi n t}{3}}\,dt\right] = \frac{3}{\pi^2 n^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2\pi n}{3}\right)$$

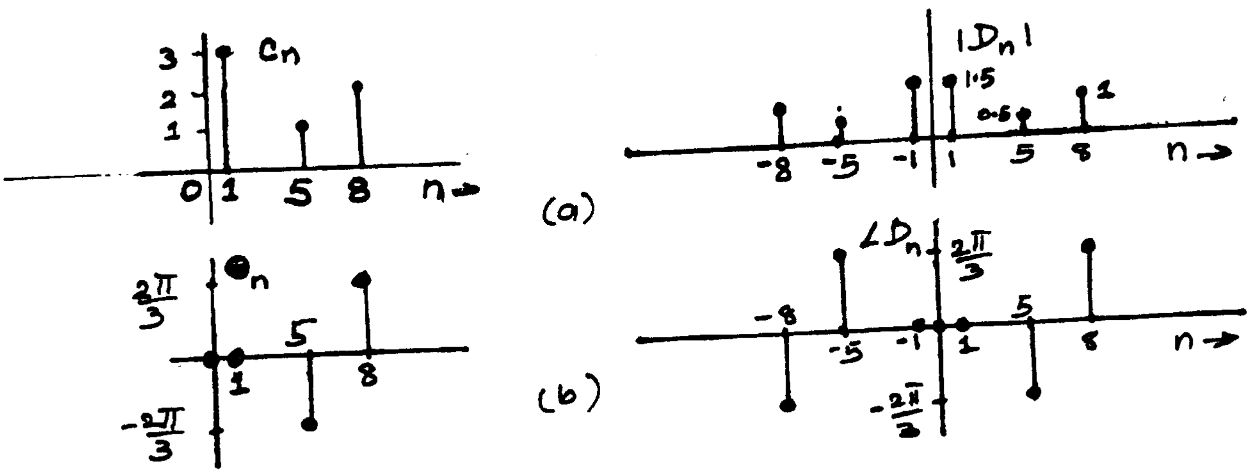


Fig. S2.9-2

2.9-2

$$g(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right)$$

For a compact trigonometric form, all terms must have cosine form and amplitudes must be positive. For this reason, we rewrite $g(t)$ as

$$\begin{aligned} g(t) &= 3\cos t + \cos\left(5t - \frac{\pi}{6} - \frac{\pi}{2}\right) + 2\cos\left(8t - \frac{\pi}{3} - \pi\right) \\ &= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t - \frac{4\pi}{3}\right) \end{aligned}$$

Figure S2.9-2a shows amplitude and phase spectra.

(b) By inspection of the trigonometric spectra in Fig. S2.9-2a, we plot the exponential spectra as shown in Fig. S2.9-2b. By inspection of exponential spectra in Fig. S2.9-2a, we obtain

$$\begin{aligned} g(t) &= \frac{3}{2}(e^{jt} + e^{-jt}) + \frac{1}{2}\left[e^{j(5t-\frac{2\pi}{3})} + e^{-j(5t-\frac{2\pi}{3})}\right] + \left[e^{j(8t-\frac{4\pi}{3})} + e^{-j(8t-\frac{4\pi}{3})}\right] \\ &= \frac{3}{2}e^{jt} + \left(\frac{1}{2}e^{-j\frac{2\pi}{3}}\right)e^{j5t} + \left(e^{-j\frac{4\pi}{3}}\right)e^{j8t} + \frac{3}{2}e^{-jt} + \left(\frac{1}{2}e^{j\frac{2\pi}{3}}\right)e^{-j5t} + \left(e^{j\frac{4\pi}{3}}\right)e^{-j8t} \end{aligned}$$

2.9-3 (a)

$$g(t) = 2 + 2\cos(2t - \pi) + \cos(3t - \frac{\pi}{2})$$
$$= 2 - 2\cos 2t + \sin 3t$$

(b) The exponential spectra are shown in Fig. S2.9-3.

(c) By inspection of exponential spectra

$$g(t) = 2 + \left[e^{(2t-\pi)} + e^{-j(2t-\pi)}\right] + \frac{1}{2}\left[e^{j(3t-\frac{\pi}{2})} + e^{-j(3t-\frac{\pi}{2})}\right]$$
$$= 2 + 2\cos(2t - \pi) + \cos\left(3t - \frac{\pi}{2}\right)$$

(d) Observe that the two expressions (trigonometric and exponential Fourier series) are equivalent.

Fig. S2.9-3

2.9-4

$$D_n = \frac{1}{T_0}\left[\int_{-T_0/2}^{T_0/2} f(t)\cos n\omega_0 t\, dt - j\int_{-T_0/2}^{T_0/2} f(t)\sin n\omega_0 t\, dt\right]$$

If $g(t)$ is even, the second term on the right-hand side is zero because its integrand is an odd function of t. Hence, D_n is real. In contrast, if $g(t)$ is odd, the first term on the right-hand side is zero because its integrand is an odd function of t. Hence, D_n is imaginary.