Microwave Engineering

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Electronics
5th

EID MUBARIK AND CONGRATS TO ALL PEOPLE WHO PASSED SUPPLY

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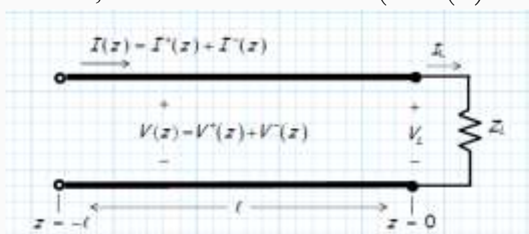
Incident, Reflected, and Absorbed Power

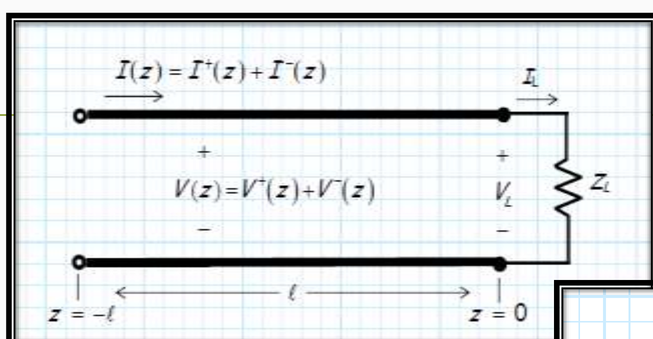
We have discovered that **two waves propagate** along a transmission line, one in each direction (V + (z) and V - (z)



How much power flows along a transmission line, and where does that power go?

We can answer that question by determining the power **absorbed** by the **load**!

Recall Signal And System



$$P_{abs} = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\}$$

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$$= \frac{1}{2}\text{Re}\{V(z=0)I(z=0)^*\}$$

$$= \frac{1}{2Z_0}\text{Re}\left\{\left(V_0^+\left[e^{-j\beta 0} + \Gamma_0 e^{+j\beta 0}\right]\right)\left(V_0^+\left[e^{-j\beta 0} - \Gamma_0 e^{+j\beta 0}\right]\right)^*\right\}$$

$$= \frac{|V_0^+|^2}{2Z_0}\text{Re}\left\{1 - (\Gamma_0^* - \Gamma_0) - |\Gamma_0|^2\right\}$$

$$= \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma_0|^2\right)$$

Can we Solve
it More ?

Solving Equation

Yeh Kahan
say agaya?

- Sure We can

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma_0|^2\right) = \frac{|V_0^+|^2}{2Z_0} - \frac{|V_0^+\Gamma_0|^2}{2Z_0} = \frac{|V_0^+|^2}{2Z_0} - \frac{|V_0^-|^2}{2Z_0}$$

Playing With Equation

Divide The Equation in two parts

Pabs= Pinc+ Preal

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0} - \frac{|V_0^-|^2}{2Z_0}$$

P*inc*

We say that this wave is **incident** on the load:

$$P_{inc} = P_+ = \frac{|V_0^+|^2}{2Z_0}$$

P*ref*

$$P_{ref} = P_{-} = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L V_0^+|^2}{2Z_0}$$

Shabash Bacha Log ??

$$P_{ref} = P_{-} = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L V_0^+|^2}{2Z_0} = |\Gamma_L|^2 \frac{|V_0^+|^2}{2Z_0} = |\Gamma_L|^2 P_{inc}$$

Concluding : The Boring Power

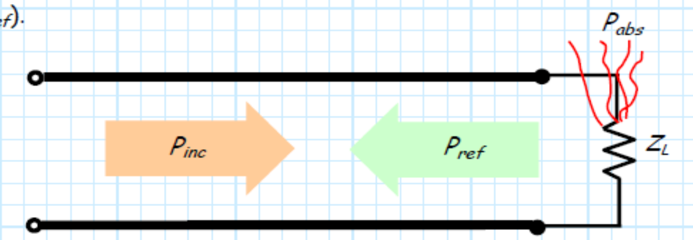
$$P_{abs} = \frac{|V_0^+|^2}{2Z_0} - \frac{|V_0^-|^2}{2Z_0} = P_{inc} - P_{ref}$$

g, we find:

$$P_{inc} = P_{abs} + P_{ref}$$

Summing it all…

It says that power flowing **toward** the load (P_{inc}) is either **absorbed** by the load (P_{abs}) or **reflected** back from the load (P_{ref}).



When Reflection Coefficient is 1

1. $|\Gamma_L|^2 = 1$

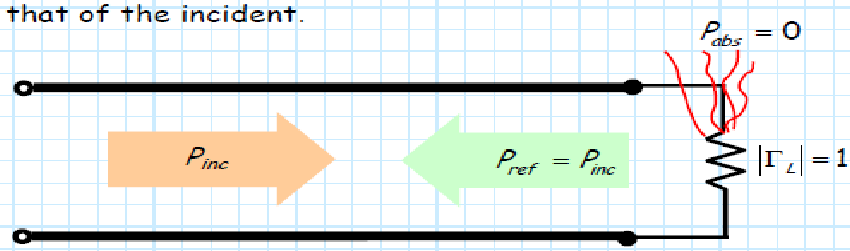
For this case, we find that the load absorbs **no power**!

$$P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) = P_{inc}\left(1 - 1\right) = 0$$

Likewise, we find that the reflected power is **equal** to the incident:

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (1) P_{inc} = P_{inc}$$

In this case, **no power** is absorbed by the load. **All** of the incident power is **reflected**, so that the reflected power is **equal** to that of the incident.



When Reflection : Coefficient is Zero

2. $|\Gamma_L| = 0$

For this case, we find that there is **no reflected power**!

$$P_{ref} = |\Gamma_L|^2 P_{inc} = (0) P_{inc} = 0$$

Likewise, we find that the absorbed power is **equal** to the incident:

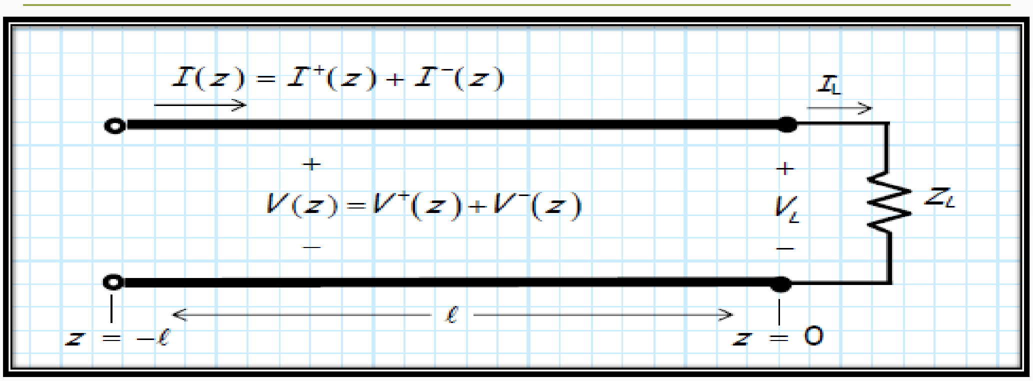
$$P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) = P_{inc}(1 - 0) = P_{inc}$$

Note these two results are completely **consistent**—by conservation of energy, if one is true the other **must** also be:

Questions

- What will Happen if reflection coefficient is greater than 1
- Can Reflection Coefficient can be greater than 1

Can I calculate Zin?



Calculating Zin

- We Know any Impedance is ratio of Voltage and Current so for Zin

$$Z_{in} = Z(z=-\ell) = \frac{V(z=-\ell)}{I(z=-\ell)}$$

Step 2: Determining Voltage and Current

$$V(z=-\ell) = V_0^+\left[e^{+j\beta\ell} + \Gamma_0 e^{-j\beta\ell}\right]$$

$$I(z=-\ell) = \frac{V_0^+}{Z_0}\left[e^{+j\beta\ell} - \Gamma_0 e^{-j\beta\ell}\right]$$

Therefore:

$$Z_{in} = \frac{V(z=-\ell)}{I(z=-\ell)} = Z_0\left(\frac{e^{+j\beta\ell} + \Gamma_0 e^{-j\beta\ell}}{e^{+j\beta\ell} - \Gamma_0 e^{-j\beta\ell}}\right)$$

Step 3 : Calculating Impedance at Any Point

- Recall …

For Loss Less transmission Line , Propagation factor is

Putting Values and Finalizing

$$Z_{in} = Z_0 \left(\frac{Z_L \cos \beta\ell + j\, Z_0 \sin \beta\ell}{Z_0 \cos \beta\ell + j\, Z_L \sin \beta\ell} \right)$$

$$= Z_0 \left(\frac{Z_L + j\, Z_0 \tan \beta\ell}{Z_0 + j\, Z_L \tan \beta\ell} \right)$$

Finalizing Values

Resource : Ahsan's Brain Xray

Special Cases

Now let's look at the Z_{in} for some important **load** impedances and **line lengths**.

→ You should commit these results to **memory**!

Case 1

1. $\ell = \lambda/2$

If the length of the transmission line is exactly **one-half** wavelength ($\ell = \lambda/2$), we find that:

$$\beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

meaning that:

$$\cos\beta\ell = \cos\pi = -1 \qquad \text{and} \qquad \sin\beta\ell = \sin\pi = 0$$

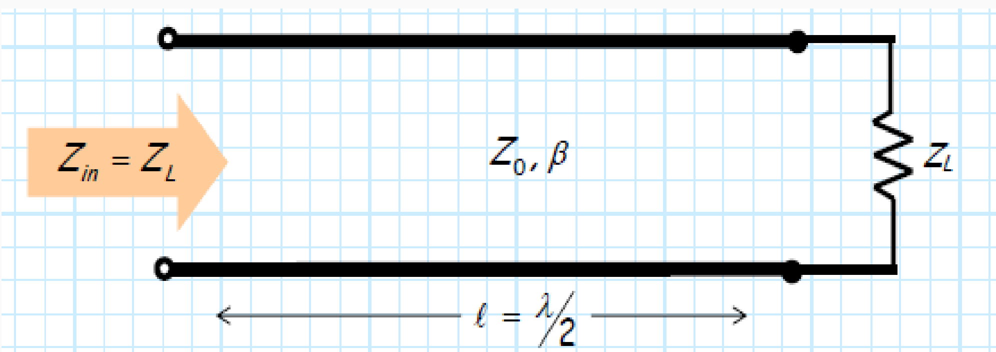
Case 1… Cont

and therefore:

$$\begin{aligned} Z_{in} &= Z_0\left(\frac{Z_L \cos\beta\ell + j\,Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\,Z_L \sin\beta\ell}\right) \\ &= Z_0\left(\frac{Z_L\,(-1) + j\,Z_L\,(0)}{Z_0\,(-1) + j\,Z_L\,(0)}\right) \\ &= Z_L \end{aligned}$$

In other words, if the transmission line is precisely **one-half wavelength** long, the **input** impedance is equal to the **load** impedance, **regardless** of Z_0 or β.

Zin= Zl



Case 2

2. $\ell = \lambda/4$

If the length of the transmission line is exactly **one-quarter** wavelength ($\ell = \lambda/4$), we find that:

$$\beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

meaning that:

$$\cos\beta\ell = \cos\pi/2 = 0 \qquad \text{and} \qquad \sin\beta\ell = \sin\pi/2 = 1$$

Case 2 .. Cont

and therefore:

$$Z_{in} = Z_0 \left(\frac{Z_L \cos\beta\ell + j\, Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\, Z_L \sin\beta\ell} \right)$$

$$= Z_0 \left(\frac{Z_L\,(0) + j\, Z_0\,(1)}{Z_0\,(0) + j\, Z_L\,(1)} \right)$$

$$= \frac{(Z_0)^2}{Z_L}$$

In other words, if the transmission line is precisely **one-quarter wavelength** long, the **input** impedance is **inversely** proportional to the **load** impedance.

Case III Zl= Zo

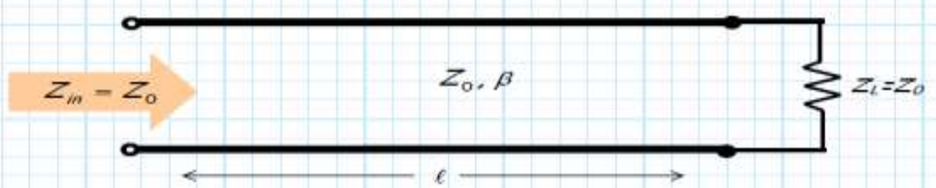
If the load is **numerically equal** to the characteristic impedance of the transmission line (a real value), we find that the input impedance becomes:

$$Z_{in} = Z_0 \left(\frac{Z_L \cos\beta\ell + j\, Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\, Z_L \sin\beta\ell} \right)$$

$$= Z_0 \left(\frac{Z_0 \cos\beta\ell + j\, Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\, Z_0 \sin\beta\ell} \right)$$

$$= Z_0$$

In other words, if the **load** impedance is equal to the transmission line **characteristic** impedance, the **input** impedance will be likewise be equal to Z_0 regardless of the transmission line length ℓ.



Case 4

4. $Z_L = j X_L$

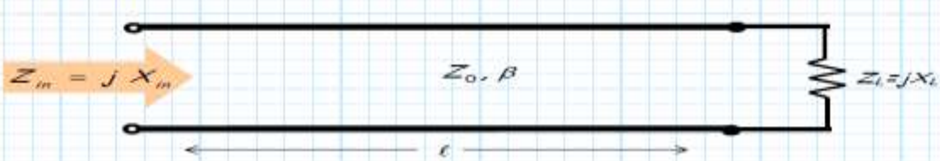
If the load is **purely reactive** (i.e., the resistive component is zero), the input impedance is:

Case 4 … Cont

$$
\begin{aligned}
Z_{in} &= Z_0\left(\frac{Z_L\cos\beta\ell + j\,Z_0\sin\beta\ell}{Z_0\cos\beta\ell + j\,Z_L\sin\beta\ell}\right)\\
&= Z_0\left(\frac{j\,X_L\cos\beta\ell + j\,Z_0\sin\beta\ell}{Z_0\cos\beta\ell + j^2\,X_L\sin\beta\ell}\right)\\
&= j\,Z_0\left(\frac{X_L\cos\beta\ell + Z_0\sin\beta\ell}{Z_0\cos\beta\ell - X_L\sin\beta\ell}\right)
\end{aligned}
$$

Case 4 .. Cont

In other words, if the load is purely reactive, then the input impedance will **likewise** be purely reactive, **regardless** of the line length ℓ.



Note that the **opposite** is **not** true: even if the load is **purely resistive** ($Z_L = R$), the input impedance will be **complex** (both resistive and reactive components).

Case 5

5. $\ell \ll \lambda$

If the transmission line is **electrically small**—its length ℓ is small with respect to signal wavelength λ--we find that:

$$\beta\ell = \frac{2\pi}{\lambda}\ell = 2\pi\frac{\ell}{\lambda} \approx 0$$

and thus:

$$\cos\beta\ell = \cos 0 = 1 \qquad \text{and} \qquad \sin\beta\ell = \sin 0 = 0$$

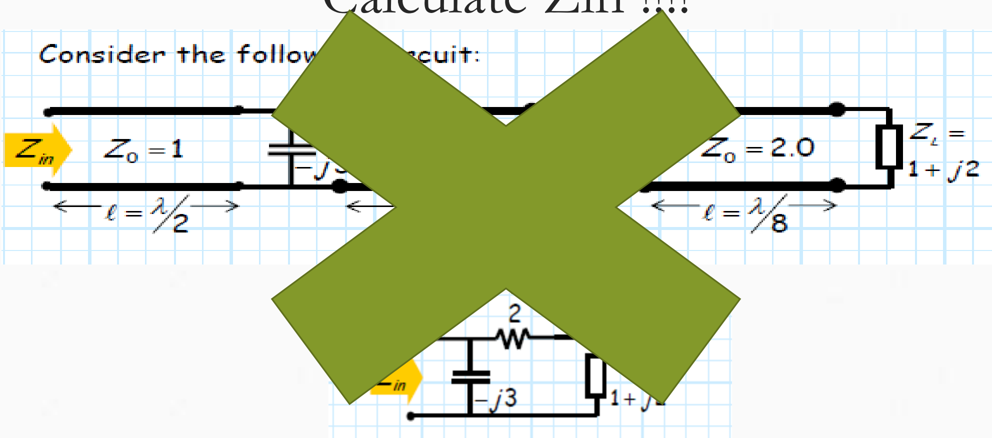
Case 5 Contd

so that the input impedance is:

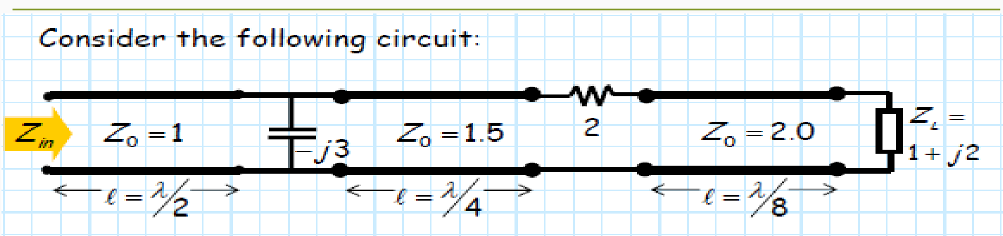
$$\begin{aligned} Z_{in} &= Z_0\left(\frac{Z_L \cos\beta\ell + j\,Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\,Z_L \sin\beta\ell}\right) \\ &= Z_0\left(\frac{Z_L\,(1) + j\,Z_L\,(0)}{Z_0\,(1) + j\,Z_L\,(0)}\right) \\ &= Z_L \end{aligned}$$

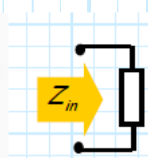
In other words, if the transmission line length is much smaller than a wavelength, the **input** impedance Z_{in} will **always** be equal to the **load** impedance Z_L.

Calculate Zin !!!!

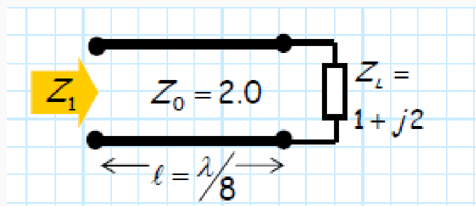


So now Calculate





Step 1



we find that Z_1 is :

$$Z_1 = Z_0 \left(\frac{Z_L \cos\beta\ell + j\, Z_0 \sin\beta\ell}{Z_0 \cos\beta\ell + j\, Z_L \sin\beta\ell} \right)$$

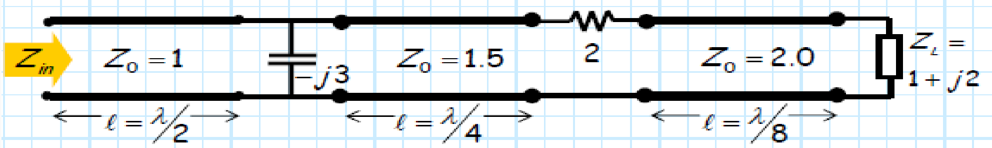
$$= 2\left(\frac{(1+j2)\cos\left(\pi/4\right) + j\, 2 \sin\left(\pi/4\right)}{2\cos\left(\pi/4\right) + j(1+j2)\, \sin\left(\pi/4\right)} \right)$$

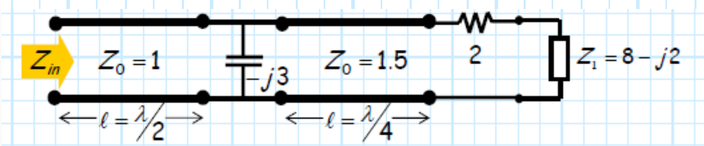
$$= 2\left(\frac{1+j4}{j} \right)$$

$$= 8 - j2$$

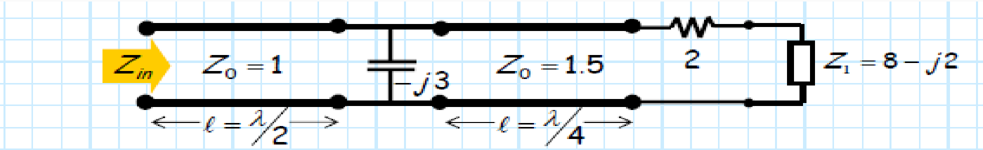
Step 2: Redraw Circuit

Consider the following circuit:



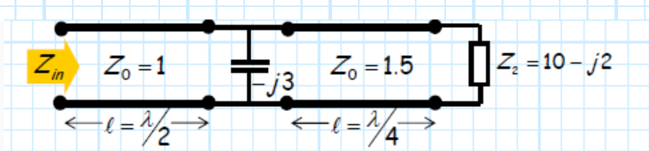


Step 3

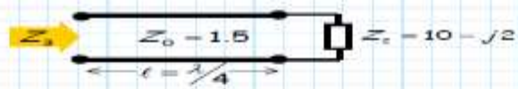


Note the resistor is in **series** with impedance Z_1. We can **combine** these two into one impedance defined as Z_2:

$$Z_2 = 2 + Z_1 = 2 + (8 - j2) = 10 - j2$$



Step 4



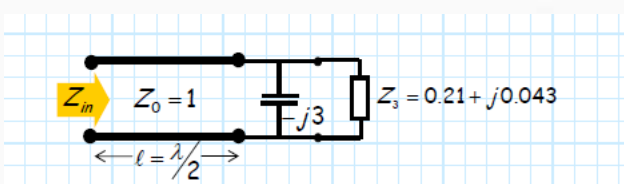
Note that this transmission line is a **quarter wavelength** ($\ell = \lambda/4$). This is one of the **special** cases we considered earlier!

The input impedance Z_3 is:

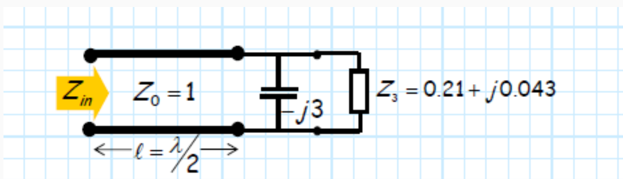
$$\begin{aligned} Z_3 &= \frac{Z_0^2}{Z_L} \\ &= \frac{Z_0^2}{Z_2} \\ &= \frac{1.5^2}{10 - j2} \\ &= 0.21 + j0.043 \end{aligned}$$

Step 5

- Redrawing Circuit



Step 6



$$= \frac{-j3(0.21 + j0.043)}{-j3 + 0.21 + j0.043}$$

$$= 0.22 + j0.028$$

DRAW